Appendix to
"Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective"

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1 Appendix: Numerical Solution and Accuracy

This appendix describes the numerical solution of the model introduced in Section 3 of Guvenen (2005) and related accuracy issues. Let $\boldsymbol{\Upsilon}$ denote the aggregate state (K, B, Z) throughout this appendix. Solving the model amounts to finding the following functions which are part of the stationary recursive equilibrium:

1. Value functions $(V^i(\omega; \boldsymbol{\Upsilon}))$ and decision rules: $b^i(\omega; \boldsymbol{\Upsilon})$ for each agent $i = h, n$, and $s'(\omega; \boldsymbol{\Upsilon})$.
2. Equilibrium bond pricing function, $q(\boldsymbol{\Upsilon})$, which clears the bond market.
3. Equilibrium laws of motion $\Gamma_K(\boldsymbol{\Upsilon})$, $\Gamma_B(\boldsymbol{\Upsilon})$ consistent with individual decision rules.

Note that there is an interdependence between the functions in 1 to 3 above. There are a number of nonlinear functional equations to solve in order to obtain these functions, so instead of attempting to solve them simultaneously, we use an iterative algorithm.

Specifically, we first solve each agent's dynamic programming problem with initial guesses for $q(\boldsymbol{\Upsilon})$ and laws of motion. Note that $R^S(K, Z)$ and $W(K, Z)$ are easily determined by the FOCs of the firm's problem. Moreover, to our knowledge, this is the first attempt to numerically solve a dynamic program with Epstein-Zin preferences. Then we use the decision rules to find a bond pricing function which clears the market and update the old value of $q(\boldsymbol{\Upsilon})$. Similarly, we update the laws of motion as will be described below. We go back to the agent's problem and solve it with the updated values of the equilibrium aggregate functions and continue the procedure until convergence. The details of the algorithm are as follows:

Step 0: *Initialization:*

(a) Choose a grid for individual wealth levels: ω^h, ω^n. We used 80 grid points for each wealth variable for the baseline case discussed in the text; using as few as 60 points and as many as 100 points did not noticeably affect the results. There is not much curvature in the equilibrium functions in K direction, so eight equally-spaced grid points gave sufficiently accurate results. On the other hand, the bond price exhibits substantial variation in B direction close to the borrowing constraints, so we took 30 grid points. We chose the locations of the grid points corresponding to Chebyshev roots which oversamples near boundaries.

(b) Take an initial guess for $q(\boldsymbol{\Upsilon})$, $\Gamma_K(\boldsymbol{\Upsilon})$ and $\Gamma_B(\boldsymbol{\Upsilon})$: we set $q^0(\boldsymbol{\Upsilon})$ such that neither asset dominates the other in return state-by-state. For $\Gamma_K^0(\boldsymbol{\Upsilon})$, we set it equal to the equilibrium law of motion for capital obtained from a representative-agent economy with the same calibration of relevant variables, and preferences are calibrated to that of the stockholder. $\Gamma_B^0(\boldsymbol{\Upsilon})$ is set such that initially, $B' = B$.

(c) Set $V^i \equiv c > 0$, $i = h, n$, for some constant c.

Step 1: *Solve each agent's dynamic problem:*

(a) Let l and j index grid points and iteration number respectively. For each grid point $(\omega_l; \boldsymbol{\Upsilon}_l)$ we find the optimal consumption and portfolio choice for the individual. The portfolio choice of the stockholder is a two-dimensional maximization problem with a very flat objective function given the small equity premium. The standard optimization packages which rely on Jacobian or Hessian matrix information, such as the "npsol" subroutine of the NAG library, or similar routines from the IMSL library fail very frequently. Hence, instead we used a constrained optimization algorithm based on the one described in Krusell and Smith (1997), which is not very fast but is very robust. To evaluate the value function off the grid points we use interpolation methods. One advantage of Epstein-Zin preferences is that without borrowing constraints the value function is linear in individual wealth. Our experience is that, in our model, it is also almost linear except in the close neighborhood of the constraint. So we were able use linear interpolation in ω direction, and we used cubic spline interpolation in K and B directions.

(b) After decision rules are obtained, we apply Howard's policy iteration algorithm to speed up convergence. This amounts to updating the value function by assuming that the agent uses the same decision rule for t periods, where we used $t = 20$.

(c) We iterate on a-b until the maximum percentage deviation in each decision rule is less than 10^{-5} for the stockholder and 10^{-6} for the non-stockholder.

Step 2: *Update the bond pricing function:* We clear the bond market following the algorithm described in Krusell and Smith (1997). In iteration j, at each grid point for current state $\boldsymbol{\Upsilon}_l$, we want to find the new bond price $q^j(\boldsymbol{\Upsilon}_l)$ which clears the markets today, when agents take $q^{j-1}(\boldsymbol{\Upsilon})$ to apply to all future dates. More specifically, we first solve the following maximization problem for the stockholder and with $s' \equiv 0$ for the non-stockholder:

$$\begin{aligned} J(\omega; \boldsymbol{\Upsilon}, \widehat{q}) &= \max_{b',\ s'} \left((1-\beta)(C)^{\varphi} + \beta \left(E_t \left[V(\omega'; \boldsymbol{\Upsilon}')^{\alpha} \right] \right)^{\frac{\varphi}{\alpha}} \right)^{\frac{1}{\varphi}} \\ & \quad s.t \\ C + \widehat{q} \times b' + s' &\leq \omega + W(K, Z) \\ & \quad \text{and equations 2 to 5 in the text.} \end{aligned}$$

Note that this is not a functional equation. This problem will give rise to bond holding rules $f_B^h(\omega; \boldsymbol{\Upsilon}, \widehat{q})$ and $f_B^n(\omega; \boldsymbol{\Upsilon}, \widehat{q})$ as a function of the current bond price $\widehat{q}$. Then, at each grid point $\boldsymbol{\Upsilon}_l$, we search over the bond price $\widehat{q}$ to find q_l^* such that the bond market clears: $\text{ABS}(\lambda \times f_B^h(\omega; \boldsymbol{\Upsilon}, q_l^*) + (1-\lambda) \times f_B^n(\omega; \boldsymbol{\Upsilon}, q^*)) < 10^{-8}$. We set $q^j(\boldsymbol{\Upsilon}_l) = q^*(\boldsymbol{\Upsilon}_l)$.

Step 3: Now we update the laws of motion using the updated decision rules: $K' = \Gamma_K^j(\boldsymbol{\Upsilon}) =$

$\lambda s^j\left(\omega^h, \boldsymbol{\Upsilon}\right)$ where $\omega^h = \left(K(1+R^e(K,Z)) - B\right)/\lambda$, and $B' = \Gamma_B^j\left(\boldsymbol{\Upsilon}\right) = (1-\lambda) b^{j,n}\left(\omega^n, \boldsymbol{\Upsilon}\right)$ where $\omega^n = B/\left(1-\lambda\right)$.

Step 4: Iterate on steps 1 to 3 until convergence. We require the maximum deviation in consecutive updates to be less than 10^{-6} for bond pricing function, and 10^{-5} for the aggregate laws of motion.

We then simulate the model through time and assume that the economy reaches a stationary distribution after 3,000 periods. All the statistics in the paper are averages over 60,000 periods of simulation.

2 Data Appendix

This appendix describes the data and the construction of variables for the joint distribution of wealth and consumption reported in Figure 1 and Table 2. The data is from the 1984 wave of the Panel Study of Income Dynamics and its wealth supplement. We choose the 1984 wave because it is one of the four waves (along with 1989, 1994 and 1999) which include the wealth supplement used to construct net worth and financial asset variables. Food data (used in the construction of consumption) is missing in the 1989 wave rendering it unusable. At the time of the writing of the first draft of this paper, the latter two waves came only in early release form with little documentation. Our attempts to construct consistent definitions of variables proved problematic. Also, due to increasing reliance on secondary respondents when the household head could not be contacted (after 1993) data quality seems to have been negatively affected (See Haider (2001) for a discussion).

The basic economic unit in all the calculations is a household. The definition of net worth and financial wealth is the same as given in footnote 6. These definitions also correspond to Wolff's (2000) variables from the Survey of Consumer Finances making comparison easier. For consumption we take expenditures of non-durables and services (denoted C_{ns}) which is the measure used by Euler equation studies discussed in the text. We construct the CEX measure of C_{ns} that we take as benchmark following Attanasio and Weber (1995) which is also very close to the National Income and Product Accounts definition. We exclude durables (vehicle purchases, household furnishings and equipment), apparel and education expenses since they are likely to have a significant durable component.

One problem with using PSID is that it does not include a comprehensive measure of consumption but rather has food expenditures (the sum of food at home, food away from home and the value of food stamps), and the rent value. On the other hand the Consumer Expenditure Survey has high quality consumption data but no detailed information on wealth. Using these two variables we construct a proxy for non-durables and services. Note that we are interested in the distribution of consumption rather than its absolute level, so we can construct a reasonable proxy as long as our variables (rent and food) constitute a reasonably fixed fraction of total consumption

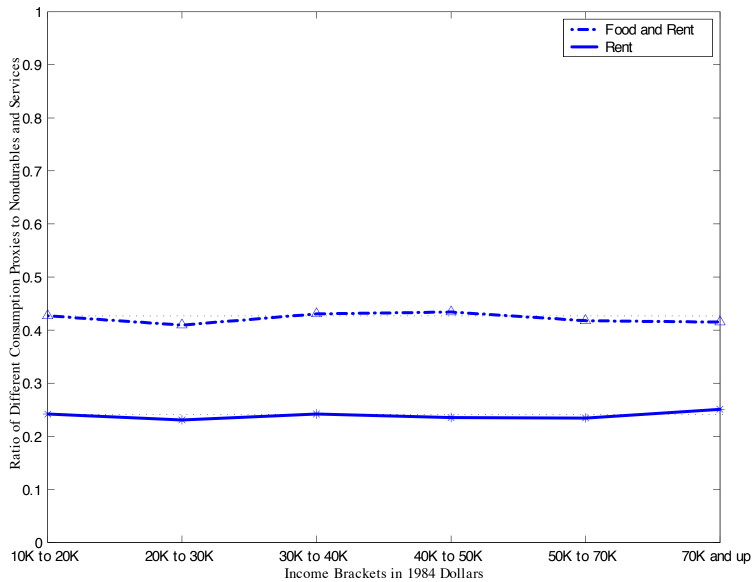

Figure 1: The Ratio of Consumption Proxies to Non-Durables and Services from the CEX, 2000

for most households. Figure 1 plots the ratio of rent to C_{ns} for different income brackets from the CEX. Rent accounts for about 23 to 25 percent of C_{ns} and is relatively stable across different household groups. Second, together with food expenditures (the dashed line) they account for 41 to 43 percent of total and this ratio is also pretty much fixed across income groups. We use this second measure to calculate the corresponding proxy from the PSID.

One last issue is about the calculation of a rental equivalent for homeowners in PSID. To obtain the rental equivalent from the reported house values for homeowners we compute the user cost of housing, $C = [(1-\tau)(i+\tau_p) - \pi + \delta]\, V \equiv \Theta V$, where τ is the personal income tax rate, τ_p is the property tax, π is capital gains on house value, δ is the maintenance cost, and V is the house value (c.f., Hendershott and Slemrod, 1983). The Bureau of Economic Analysis (BEA) releases annual data that measure the value of the stock of owner-occupied nonfarm housing as well as the imputed rent for owner-occupied non-farm housing. (The imputed rent is calculated using actual rents on comparable dwellings both from the Census of Housing Survey and the CPI housing survey.) Dividing the two numbers yields a measure of Θ equal to 8.8 percent which we use to calculate the rental equivalents from house values. To see if possible variation in Θ across income levels might bias the results, we also constructed the same consumption proxy using only renters' information. In this case, the share of consumption of the top 20 percent was 34.2 percent. We also tried using total consumption expenditures (instead of non-durables and services) as the measure of consumption. In this case the share of consumption of the top 20 percent increased from 30.3 percent to 33.6 percent.

Table 1: Estimation of the Log-Linearized Euler Equation (6) with Post-War U.S. Data with and without Correcting for Time Variation

$Var_t(\Delta c_{t+1})$ included?	**No**	**Yes**			
N (quarters)	—	4	8	12	16
EIS estim. with $\mathbf{Z}_1$	0.11	0.29	0.63	0.48	0.41
(t-stat)	(0.94)	(1.21)	(1.67)	(1.94)	(1.49)
EIS estim. with $\mathbf{Z}_2$	-0.09	0.39	0.57	0.40	0.51
(t-stat)	(-0.97)	(1.82)	(1.92)	(2.28)	(2.12)

3 Appendix: Replicating Hall (1988) with Time-varying Conditional Variances

To gain some insight into how serious this problem is, we conducted the following simple experiment with actual data. We re-ran Hall's regression, closely matching the definition of variables, time period and construction of variables to existing studies. As a first step, we calculate the correlation of the variance of consumption growth with once lagged interest rate, and surprisingly it is -0.2, closely matching that in the model. This also provides some further support to the model in terms of capturing the rich dynamics of data.

The consumption measure is the real quarterly consumption of non-durables from the National Income and Products Account covering 1951.1 to 1984.4. The interest rate is the 3-month T-bill rate from the FRED database, deflated by the corresponding NIPA non-durable consumption prices deflator. Finally, $var_t(\Delta c_{t+1})$ is calculated as follows: for each quarter t, we computed the sample variance using the realized consumption changes in the next N quarters (Δc_{t+1},..., Δc_{t+N}) Results are reported for a range of values of N. The first instrument set is $Z_1 = (1,\ r^f_{t-2}, r^f_{t-3}$, $r^f_{t-4},\ \Delta c_{t-2}, \Delta c_{t-3}, \Delta c_{t-4})$ and the second one is $Z_2 = (Z_1, i_{t-2}, i_{t-3}, i_{t-4})$ where i_t denotes the inflation rate between t and $t+1$.

Table 12 presents the results. When the intercept k, is assumed to be constant (column 1) the EIS is estimated to be 0.11 and insignificant, in line with most of the previous literature. However, with $var_t(\Delta c_{t+1})$ appropriately included in the rest of the table, all of a sudden the estimates of EIS jump to around 0.4. Repeating the same experiment with a larger instrument set only strengthens this result, if anything we get more precise estimates which cluster around 0.4 to 0.5.

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