ONLINE APPENDIX

for

Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation

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A Model Details and Additional Equations

A.1 Social Security Pension System

When an individual retires at age R, she starts receiving social security income $y^R(\kappa, e)$ that depends on her type κ in the following way:

$$y^R(\kappa, e) = \Phi(\kappa, e)\,\overline{y}, \tag{A.1}$$

where Φ is the replacement ratio. The replacement ratio is progressive and given by

$$\Phi(\kappa, e) = \begin{cases} 0.9\frac{y_1^R(\kappa,e)}{\overline{y}_1^R} & \text{if } \frac{y_1^R(\kappa,e)}{\overline{y}_1^R} \leqslant 0.3 \\ 0.27 + 0.32\left(\frac{y_1^R(\kappa,e)}{\overline{y}_1^R} - 0.3\right) & \text{if } 0.3 < \frac{y_1^R(\kappa,e)}{\overline{y}_1^R} \leqslant 2 \\ 0.81 + 0.15\left(\frac{y_1^R(\kappa,e)}{\overline{y}_1^R} - 2\right) & \text{if } 2 < \frac{y_1^R(\kappa,e)}{\overline{y}_1^R} \leqslant 4.1 \\ 1.13 & \text{if } 4.1 < \frac{y_1^R(\kappa,e)}{\overline{y}_1^R} \end{cases} \tag{A.2}$$

where $y_1^R(\kappa, e)$ is the average efficiency units over lifetime that an individual of type κ gets conditional on having a given $e_R = e$:

$$y_1^R(\kappa, e_R) = \frac{1}{R}\int_{h<R,a,\mathbf{S}} y_h(\kappa, e)\, d\Gamma(h, a, \mathbf{S}). \tag{A.3}$$

The vector $\mathbf{S} = (\overline{z}, \mathbb{I}, \kappa, e)$ is the vector of exogenous states of an individual, and the integral is taken with respect to the stationary distribution (Γ) of individuals so that e_R is the one given on the left-hand side. Finally, $\overline{y}_1^R$ is the average of $y_1^R(\kappa, e)$ across κ and e. The term SSP denotes the aggregate value of "social security pension" payments:

$$SSP \equiv \int_{h\geqslant R,a,\mathbf{S}} y^R(\kappa, e)\, d\Gamma(h, a, \mathbf{S}). \tag{A.4}$$

A.2 Recursive Competitive Equilibrium

Definition. *Let* $c_h(a, \mathbf{S})$*,* $\ell_h(a, \mathbf{S})$*,* $a_{h+1}(a, \mathbf{S})$*, and* $k(a, z)$ *denote the optimal decision rules and* $\Gamma(h, a, \mathbf{S})$ *denote the stationary distribution of individuals. A recursive competitive equilibrium is given by the following conditions:*

1. *Consumers maximize utility given* $p(x)$*,* $\overline{w}$*,* r*, and taxes.*
2. *The solution to the final goods producer gives the pricing function,* $p(x)$*, and wage rate,* $\overline{w}$*.*
3. $Q = \left(\int_{h,a,\mathbf{S}} (z \times k(a, z))^{\mu}\, d\Gamma(h, a, \mathbf{S})\right)^{1/\mu}$ *and* $L = \int_{h,a,\mathbf{S}} (w_h(\kappa, e)\,\ell_h(a, \mathbf{S}))\, d\Gamma(h, a, \mathbf{S})$*, where* $\log w_h(\kappa, e) = \kappa + g(h) + e$*.*

4. *The bond market clears:*

$$0 = \int_{h,a,\mathbf{S}} (a - k(a,z))\, d\Gamma(h,a,\mathbf{S})\,. \tag{A.5}$$

5. *The government budget balances. The revenue raised by taxes on labor, consumption, bequests, and capital income or wealth equals government consumption,* G, *plus pension payments to retirees, SSP:*

$$\begin{aligned} G + SSP = {} & \tau_k \int_{h,a,\mathbf{S}} (ra + \pi(a,z))\, d\Gamma(h,a,\mathbf{S}) \\ & + \tau_a \int_{h,a,\mathbf{S}} (a)\, d\Gamma(h,a,\mathbf{S}) \\ & + \tau_\ell \int_{h<R,a,\mathbf{S}} (\overline{w} w_h(\kappa,e)\, \ell_h(a,\mathbf{S}))\, d\Gamma(h,a,\mathbf{S}) \\ & + \tau_c \int_{h,a,\mathbf{S}} c_h(a,\mathbf{S})\, d\Gamma(h,a,\mathbf{S})\,, \\ & + \tau_b \int_{h,a,\mathbf{S}} (1 - s_{h+1})\, a_{h+1}(a,\mathbf{S})\, d\Gamma(h,a,\mathbf{S})\,, \end{aligned} \tag{A.6}$$

where $\tau_a \equiv 0$ *in the capital income tax economy and* $\tau_k \equiv 0$ *in the wealth tax economy, and SSP is given in equation (A.4).*

A.3 Formulas for Welfare Analyses

A.3.1 Formulas for Section 5.2

The formulas that define CE_1 and $\overline{\mathsf{CE}}_2$, introduced in Section 5.2, are as follows. We compute CE_1 for an h-year-old individual in state $\mathcal{S} = (a, \mathbf{S})$ as the percentage change in consumption at all future dates and states required to make her indifferent between the stationary equilibria of the two economies:

$$V_h^{US}\left((1 + \mathsf{CE}_1(h,\mathcal{S})) \times c_h^{US}(\mathcal{S}), \ell_h^{US}(\mathcal{S})\right) = V_h^{RN}\left(c_h^{RN}(\mathcal{S}), \ell_h^{RN}(\mathcal{S})\right), \tag{A.7}$$

where V_h is the lifetime value function and (c, ℓ) are the consumption and leisure allocations starting from state $(h, \mathcal{S})$, and the superscripts indicate the relevant economy (e.g., US versus RN). At the aggregate level, the main measure we will look at is the welfare change for newborns, which is obtained by integrating over the stationary distribution in the benchmark economy $(\Gamma_{h=1}^{US}(\mathcal{S}))$:

$$\overline{\mathsf{CE}}_1 \equiv \sum_{\mathcal{S}} \Gamma^{US}(1,\mathcal{S}) \times \mathsf{CE}_1(1,\mathcal{S})\,. \tag{A.8}$$

Using equation 1 and the fact that $u(c,\ell) = \frac{\left(c^{\gamma}(1-\ell)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$, we can compute $\mathsf{CE}_1(h,\mathcal{S})$ directly from the value functions: $1 + \mathsf{CE}_1(h,\mathcal{S}) = \left(\frac{V_h^{R}(\mathcal{S}) - B_h^{US}(\mathcal{S})}{V_h^{US}(\mathcal{S}) - B_h^{US}(\mathcal{S})}\right)^{1/\gamma(1-\sigma)}$, where $V_h^{US}(\mathcal{S})$ is the value function of an agent of age h at state $\mathcal{S}$ and $B_h(\mathcal{S})$ is the expected discounted value of the utility payoff of bequests (thus, $V_h^{US}(\mathcal{S}) - B_h^{US}(\mathcal{S})$ gives the value coming just from consumption and leisure).

$\overline{CE}_2$ measures the fixed proportional consumption transfer to all newborn individuals in the US benchmark economy such that average utility is equal to that in the tax-reform economy. For the RN reform, it reads

$$\sum_{\mathcal{S}} \Gamma^{US}(1,\mathcal{S}) \cdot V_1^{US}\left(\left(1+\overline{CE}_2\right) c_1^{US}(\mathcal{S}), \ell_1^{US}(\mathcal{S})\right) = \sum_{\mathcal{S}} \Gamma^{RN}(1,\mathcal{S}) \cdot V_1^{RN}\left(c_1^{RN}(\mathcal{S}), \ell_1^{RN}(\mathcal{S})\right). \quad \text{(A.9)}$$

We can get a closed form expression for the welfare gain by first defining the average expected discounted value of the utility payoff of bequests for newborn agents:

$$\overline{B}_1^{US}(\mathcal{S}) \equiv \sum_{\mathcal{S}} \Gamma^{US}(1,\mathcal{S}) \cdot B_1^{US}(\mathcal{S}). \quad \text{(A.10)}$$

This allows us to get an expression for the welfare gain:

$$1+\overline{CE}_2 = \left(\frac{\sum_{\mathcal{S}} \Gamma^{RN}(1,\mathcal{S}) \cdot V_1^{RN}(\mathcal{S}) - \overline{B}_1^{US}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{US}(1,\mathcal{S}) \cdot V_1^{US}(\mathcal{S}) - \overline{B}_1^{US}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}}. \quad \text{(A.11)}$$

A.3.2 Formulas for the Welfare Decomposition in Section 6.3

We derive in this section the formulas for decomposing $\overline{CE}_2$. The formulas for CE_1 are analogous.

Level-Distribution Decomposition

The welfare gain from changes in consumption and leisure can be jointly decomposed into gains from changes in levels and gains from changes in the distribution.

Consumption and Leisure Level To construct this measure, define first an alternative consumption policy that takes into account just the change in the level of aggregate consumption:

$$\hat{c}_h(\mathcal{S}) \equiv \frac{C^{RN}}{C^{US}} c_h^{US}(\mathcal{S}) \qquad \text{where } C^x \equiv \sum_{h,\mathcal{S}} c_h^x(\mathcal{S})\, \Gamma^x(h,\mathcal{S}) \text{ for } x \in \{\text{US,RN}\}. \quad \text{(A.12)}$$

Similarly, define the alternative policy for leisure as

$$\hat{\ell}_h(\mathcal{S}) \equiv \frac{L^{RN}}{L^{US}} \ell_h^{US}(\mathcal{S}) \qquad \text{where } L^x \equiv \sum_{h,\mathcal{S}} \ell_h^x(\mathcal{S})\, \Gamma^x(h,\mathcal{S}) \text{ for } x \in \{\text{US,RN}\}. \quad \text{(A.13)}$$

The level gain is obtained by equating the welfare under the benchmark policies and the alternative policies defined above, while keeping constant bequests in the two economies:

$$\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}\left(\left(1+\overline{\mathsf{CE}}_2^{\mathsf{L}}\right) c_1^{\mathrm{US}}(\mathcal{S}), \ell_1^{\mathrm{US}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right)$$
$$= \sum_{\mathcal{S}} \Gamma^{\mathrm{RN}}(1,\mathcal{S}) \cdot V_1^{\mathrm{RN}}\left(\hat{c}_1(\mathcal{S}), \hat{\ell}_1(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right). \tag{A.14}$$

Given the preferences, we assume this gives

$$\begin{aligned}
1+\overline{\mathsf{CE}}_2^{\mathsf{L}} &= \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(\hat{c}_1(\mathcal{S}), \hat{\ell}_1(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B_1^{\mathrm{US}}(\mathcal{S})}}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B_1^{\mathrm{US}}(\mathcal{S})}}\right)^{\frac{1}{\gamma(1-\sigma)}} \\
&= \frac{C^{\mathsf{RN}}}{C^{\mathsf{US}}}\left(\frac{L^{\mathsf{RN}}}{L^{\mathsf{US}}}\right)^{\frac{1-\gamma}{\gamma}} \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c_1^{\mathrm{US}}(\mathcal{S}), \ell_1^{\mathrm{US}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}} \\
&= \frac{C^{\mathsf{RN}}}{C^{\mathsf{US}}}\left(\frac{L^{\mathsf{RN}}}{L^{\mathsf{US}}}\right)^{\frac{1-\gamma}{\gamma}}.
\end{aligned} \tag{A.15}$$

Consumption and Leisure Distribution The distributional gains correspond to the change in the value of agents from adjusting the policy functions while keeping the level comparable. Once again we keep the value of bequests fixed:

$$\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}\left(\left(1+\overline{\mathsf{CE}}_2^{\mathsf{D}}\right) \hat{c}_1(\mathcal{S}), \hat{\ell}_1(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right)$$
$$= \sum_{\mathcal{S}} \Gamma^{\mathrm{RN}}(1,\mathcal{S}) \cdot V_1^{\mathrm{RN}}\left(c_1^{\mathrm{RN}}(\mathcal{S}), \ell_1^{\mathrm{RN}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right). \tag{A.16}$$

Given the preferences we assume, this gives

$$\begin{aligned}
1+\overline{\mathsf{CE}}_2^{\mathsf{D}} &= \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c_1^{\mathsf{RN}}(\mathcal{S}), \ell_1^{\mathsf{RN}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(\hat{c}_1(\mathcal{S}), \hat{\ell}_1(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}} \\
&= \left[\frac{C^{\mathsf{RN}}}{C^{\mathsf{US}}}\left(\frac{L^{\mathsf{RN}}}{L^{\mathsf{US}}}\right)^{\frac{1-\gamma}{\gamma}}\right]^{-1} \times \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c_1^{\mathsf{RN}}(\mathcal{S}), \ell_1^{\mathsf{RN}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}}\right) \\
&= \frac{1}{1+\overline{\mathsf{CE}}_2^{\mathsf{L}}}\left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c_1^{\mathsf{RN}}(\mathcal{S}), \ell_1^{\mathsf{RN}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}}\right)^{\frac{1}{\gamma(1-\sigma)}} \\
&= \frac{1+\overline{\mathsf{CE}}_2^{c\ell}}{1+\overline{\mathsf{CE}}_2^{\mathsf{L}}},
\end{aligned} \tag{A.17}$$

where we define

$$1+\overline{\mathsf{CE}}_2^{c\ell} \equiv \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c_1^{\mathsf{RN}}(\mathcal{S}), \ell_1^{\mathsf{RN}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}} \tag{A.18}$$

as the joint gain from consumption and leisure. By construction we can decompose the value form consumption and leisure into the level and distributional changes:

$$1+\overline{\mathsf{CE}}_2^{c\ell} = \left(1+\overline{\mathsf{CE}}_2^{L}\right)\left(1+\overline{\mathsf{CE}}_2^{D}\right). \tag{A.19}$$

Complete Decomposition

To totally decompose the level of the consumption-equivalent welfare gain, we need to take into account the change in bequests. This is

$$\begin{aligned}
1+\overline{\mathrm{CE}}_2 &= \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{RN}}(1,\mathcal{S}) \cdot V_1^{\mathrm{RN}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}} \\
1+\overline{\mathrm{CE}}_2 &= \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c^{\mathsf{RN}}(1,\mathcal{S}), \ell^{\mathsf{RN}}(1,\mathcal{S}), b^{\mathsf{US}}(1,\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1^{\mathrm{US}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}} \times \\
&\qquad \left(\frac{\sum_{\mathcal{S}} \Gamma^{\mathrm{RN}}(1,\mathcal{S}) \cdot V_1^{\mathrm{RN}}(\mathcal{S}) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}{\sum_{\mathcal{S}} \Gamma^{\mathrm{US}}(1,\mathcal{S}) \cdot V_1\left(c_1^{\mathsf{RN}}(\mathcal{S}), \ell_1^{\mathsf{RN}}(\mathcal{S}), b_1^{\mathsf{US}}(\mathcal{S})\right) - \overline{B}_1^{\mathrm{US}}(\mathcal{S})}\right)^{\frac{1}{\gamma(1-\sigma)}} \\
1+\overline{\mathrm{CE}}_2 &= \left(1+\overline{\mathrm{CE}}_2^{c\ell}\right)\left(1+\overline{\mathrm{CE}}_2^{b}\right).
\end{aligned} \tag{A.20}$$

B Additional Tables

TABLE B.1 – *Forbes* Self-Made Index

	DESCRIPTION	FRACTION
1	Inherited fortune but not working to increase it	7.00
2	Inherited fortune and has a role managing it	4.75
3	Inherited fortune and helping to increase it marginally	5.50
4	Inherited fortune and increasing it in a meaningful way	5.25
5	Inherited small or medium-size business and made it into a 10-digit fortune	8.50
6	Hired or hands-off investor who didn't create the business	2.25
7	Self-made who got a head start from wealthy parents & moneyed background	10.00
8	Self-made who came from a middle- or upper-middle-class background	**32.00**
9	Self-made who came from a largely working-class background; rose from little to nothing	**14.50**
10	Self-made who not only grew up poor but also overcame significant obstacles	**7.75**
	Forbes's definition of self-made: Groups 8 to 10	**54.25**

Notes: Table reports *Forbes*'s categories for classifying individuals in its top-400 list, along with their share among the individuals in the list. Self-made individuals correspond to categories 8, 9, and 10.

B.1 Additional Results on the Distribution of Capital Income and Wealth

The benchmark model is consistent with the high concentration of capital income in the economy. Table B.2 shows that the concentration of capital income is higher than the concentration of wealth in the model. For instance, those in the top 1% of the wealth distribution hold 35.1% of all the wealth and 48.2% of all the capital income in the economy. Those in the top 1% of the capital income distribution hold 51.9% of all the capital income in the economy. Furthermore, the Gini coefficients of wealth and capital income are 0.78 and 0.87, respectively.

Simultaneously, capital income and wealth are highly correlated in the model. The correlation coefficient is 0.85. This is consistent with Table B.2, which shows that capital income is concentrated among the wealthiest individuals in the economy. It is also consistent with the correlation of the returns to wealth and wealth levels. In particular, for the 35-49 age group, returns are in the 5%-6.1% range in the bottom half of the wealth distribution but increase to 6.5% at the 60th percentile, 7.3% at the 95th, 8.4% at 99th, 11.4% at the 99.9th, and 12.7% at the 100th. The same patterns arise later in the life cycle of individuals but with lower levels of returns.

Table B.2 – Concentration of Capital Income and Wealth

Top x% of Wealth Dist.	Wealth Share (%)	Capital Income Share (%)	Top x% of Capital Income Dist.	Capital Income Share (%)
0.1	22.3	32.0	0.1	34.3
0.5	30.5	43.0	0.5	45.7
1	35.1	48.2	1	51.9
10	64.9	73.1	10	78.9
50	96.4	97.0	50	98.1

Notes: The table describes the concentration of the wealth and capital income distribution. The left panel reports top wealth shares and the corresponding capital income shares of individuals in the respective group of top-wealth holders. The right panel reports the capital income shares of top-capital-income earners. All numbers in percentage points.

B.2 Additional Results on the Distribution of Welfare Gains/Losses

Table B.3 – Optimal Tax Experiments: Distribution of Welfare Gains and Losses

(a) Optimal Capital Income Taxes

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	3.4	3.8	5.1	7.5	11.4	13.8	99.6	98.8	99.8	99.9	100.0	100.0
21–34	3.3	3.6	4.7	7.0	11.2	13.9	99.7	99.1	99.8	99.9	100.0	100.0
35–49	2.9	2.8	3.5	4.8	7.1	8.7	99.4	98.0	99.6	99.8	100.0	100.0
50–64	1.6	1.5	1.9	2.7	3.8	4.6	97.8	94.9	99.3	99.6	99.9	99.9
65+	0.1	0.2	0.4	0.9	1.6	1.9	94.5	96.3	99.8	99.9	99.9	99.8

(b) Optimal Wealth Taxes

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	9.4	8.3	8.3	10.1	13.9	16.3	97.5	94.6	94.3	95.8	97.9	98.8
21–34	8.7	6.8	5.8	6.4	8.0	8.6	97.6	92.9	90.0	90.2	89.7	87.0
35–49	6.3	4.1	2.4	1.6	–0.4	–2.3	93.6	80.4	71.0	64.5	52.6	42.4
50–64	2.5	1.0	–0.1	–1.2	–3.4	–5.2	74.9	62.5	52.9	45.3	34.5	27.6
65+	–0.5	–0.9	–1.3	–1.9	–3.1	–4.3	1.4	0.8	0.7	0.7	0.6	0.4

(c) Optimal Wealth Taxes with Exemption Threshold

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	9.4	8.5	8.6	10.4	14.3	17.0	97.2	93.1	92.4	95.0	97.5	98.3
21–34	8.7	6.8	5.8	6.3	7.7	8.1	97.3	91.3	86.9	87.4	87.0	83.7
35–49	6.3	3.9	2.0	0.9	–1.7	–4.3	92.4	78.7	67.6	60.5	48.2	38.4
50–64	2.6	1.1	–0.3	–1.7	–4.6	–7.0	78.7	66.3	56.4	48.0	36.2	28.9
65+	–0.3	–0.7	–1.1	–2.0	–3.7	–5.3	79.8	73.3	65.1	56.6	43.8	35.4

Notes: Each panel reports the average welfare gain (CE_1) and the share of individuals who experience a positive welfare gain (CE_1) in a given age and entrepreneurial productivity group (ranked based on the permanent component of entrepreneurial productivity $\bar{z}$) from the corresponding optimal tax experiment. The average and shares are computed with respect to the benchmark distribution. All numbers are in percentage points.

Table B.4 – Welfare Change *with Transition*: Switch to Optimal Tax System with Transition

(a) Optimal Capital Income Taxes

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	-8.8	-7.5	-4.8	0.2	8.7	13.8	4.1	8.2	17.0	27.3	71.8	99.6
21–34	-8.2	-5.9	-1.9	5.7	19.8	30.2	3.5	10.7	28.4	58.9	81.0	84.8
35–49	-6.3	-3.9	0.0	6.5	18.5	27.1	8.6	20.2	47.8	58.9	69.9	75.0
50–64	-3.1	-1.3	1.3	5.2	12.2	17.0	26.5	37.9	54.4	60.7	69.6	75.3
65+	0.6	1.2	2.2	4.0	7.0	9.1	99.6	100	100	100	100	100

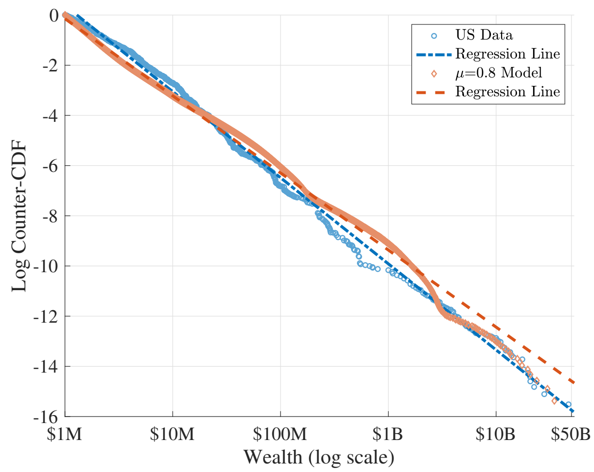
(b) Optimal Wealth Taxes

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	5.4	4.9	5.6	8.4	13.5	16.7	95.7	93.8	95.0	97.7	99.5	99.7
21–34	4.8	3.8	3.9	6.0	10.0	12.1	95.6	90.6	90.5	93.5	94.9	94.2
35–49	2.9	1.7	1.1	1.5	1.6	1.0	84.6	72.8	67.3	69.4	67.8	64.5
50–64	0.5	-0.3	-0.8	-1.1	-2.2	-3.4	59.8	50.6	44.1	42.4	38.6	35.9
65+	-0.7	-0.9	-1.1	-1.4	-2.5	-3.7	3.2	5.5	6.9	9.0	10.9	11.5

Notes: Each panel reports the average welfare gain (CE_1) and the share of individuals who experience a positive welfare gain (CE_1) in a given age and entrepreneurial productivity group (ranked based on the permanent component of entrepreneurial productivity $\bar{z}$) who are alive at the time of the corresponding tax experiment with transition. The average and shares are computed with respect to the benchmark distribution. All numbers are in percentage points.

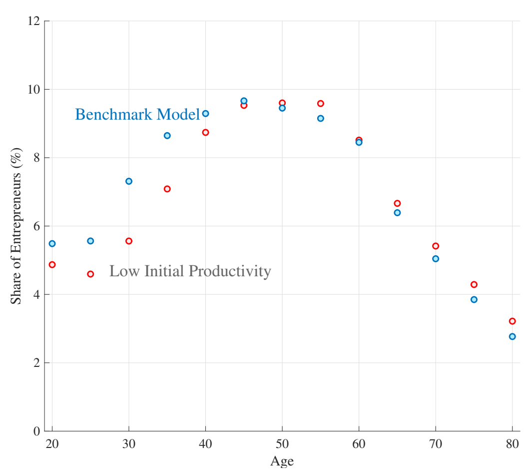
C Additional Figures

Figure C.1 – Stronger Diminishing Returns in Entrepreneurial Production, $\mu = 0.8$



Notes: The figure reports the natural logarithm of the counter-CDF of wealth above one million dollars, corresponding to the right tail of the distribution. The data for the US (in blue) come from Vermeulen (2018). The orange diamonds correspond an alternative calibration of the model with $\mu = 0.8$. Both axes are in natural logs. The horizontal axis ticks are placed at powers of 10 for readability.

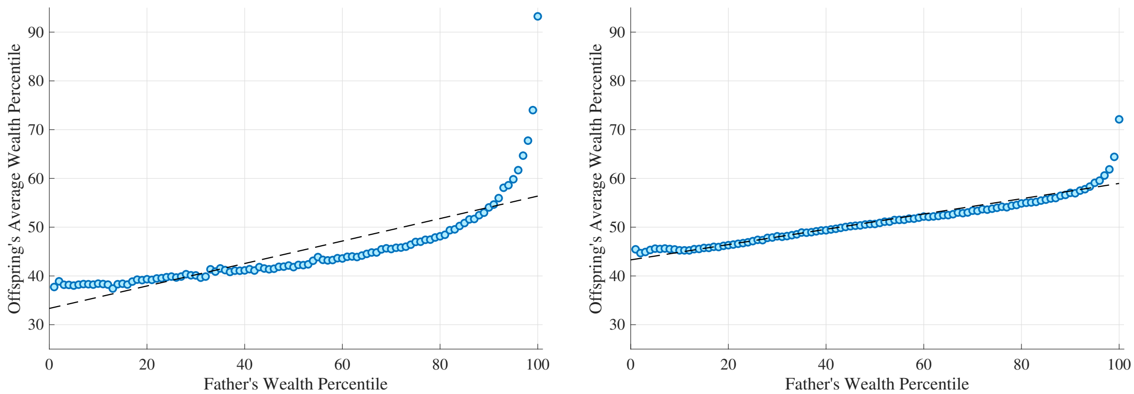
Figure C.2 – Fraction of Entrepreneurs over the Life Cycle



Notes: The figure plots the fraction of entrepreneurs, in percent, over the life cycle. An entrepreneur is defined as someone who earns more than 50% of their income from their business. The low-initial-productivity model has the same productivity shock process as in the benchmark, except that nobody starts in the fast lane ($z_{i0} = \bar{z}_i$) but those with $\bar{z}_i$ above median have a 3% probability of entering the fast lane each period.

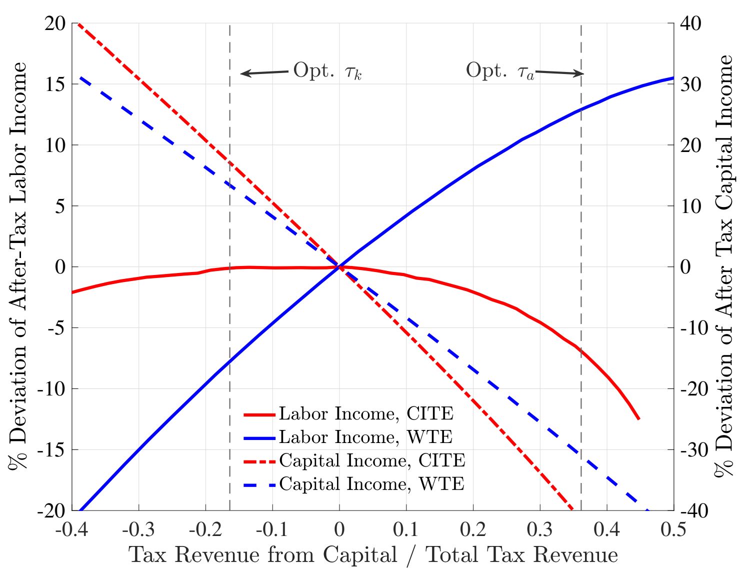
Figure C.3 – Intergenerational Rank-Rank Correlation in Wealth: Model vs. Data

(a) Baseline Model (b) Norway: Fagereng et al. (2020, Figure 11)



Notes: The figures report the rank-rank plots of wealth between fathers and their offspring for the baseline model and Norwegian data. The blue circles mark the average percentile within the cohort of the offspring of fathers in a given percentile (rank) of the wealth distribution. The dashed line corresponds to the trend line. The Norwegian data come from Fagereng et al. (2020, Figure 11).

Figure C.4 – Average After-Tax Labor and Capital Income vs. Capital Tax Revenues



Notes: The figure reports the percentage change of total labor (solid line) and capital income (dashed line) with respect to the benchmark economy for different levels capital income tax (CITE in red) or wealth tax (WTE in blue). For each level of the tax the labor income tax adjusts to balance the government's budget. Welfare gains are in percentages. Each economy is indexed by its ratio of tax revenue from capital income or wealth taxes to total revenue. Total revenue is constant across economies.

D Misallocation in the Economy

Our model economy is distorted because of the existence of financial frictions in the form of borrowing constraints. We can measure the effects of these distortions on aggregate TFP and output, following a large and growing literature that frames the discussion on misallocation in terms of various wedges, such as capital, labor, and output wedges. In particular, we follow Hsieh and Klenow (2009) and compute measures of misallocation for our model economy.

Instead of modeling and capturing the effect of a particular distortion, or distortions, we infer the underlying distortions and wedges in the economy by studying the extent to which the marginal revenue products of capital and labor differ across firms. This is based on the insight that without any distortions, the marginal revenue products of capital and labor have to be equalized across all firms.[53]

In our model, competitive final goods producers use effective capital, Q, and labor, L, in production as in (9), where Q is produced using intermediate goods as in (10). Each intermediate goods producer i produces $x_i = z_i k_i$, where z_i is i's entrepreneurial ability and k_i is capital.

TFP in the Q sector. We will first focus on the intermediate goods sector. Under the alternative capital-wedge approach, the problem of each intermediate goods producer is

$$\pi_i = \max_{k_i} p\left(z_i k_i\right) z_i k_i - \left(1+\tau_i\right)\left(R+\delta\right) k_i \ , \tag{D.1}$$

where τ_i is a firm-specific wedge. There are no collateral constraints. There is only one input and, as a result, only one wedge can be identified.

The revenue TFP in sector Q for each firm i is

$$\mathsf{TFPR}_{Q,i} \equiv \frac{p\left(x_i\right) x_i}{k_i} = \frac{1}{\mu}\left(1+\tau_i\right)\left(R+\delta\right). \tag{D.2}$$

The aggregate TFP in sector Q can be expressed as

$$\mathsf{TFP}_Q \equiv \frac{Q}{K} = \left(\int_i \left(z_i \frac{\overline{\mathsf{TFPR}_Q}}{\mathsf{TFPR}_{Q,i}}\right)^{\frac{\mu}{1-\mu}} di\right)^{\frac{1-\mu}{\mu}} , \tag{D.3}$$

where the average TFPR_Q is

$$\overline{\mathsf{TFPR}_Q} = \left(\int \frac{1}{\mathsf{TFPR}_{Q,i}} \frac{p\left(x_i\right) x_i}{p_q Q} di\right)^{-1} . \tag{D.4}$$

[53] This is the case in the monopolistic competition models, such as in Hsieh and Klenow (2009). Alternatively, in environments like the ones in Lucas (1978) and Restuccia and Rogerson (2008), in which firms feature decreasing returns to scale but produce the same homogeneous good, the marginal products of capital and labor have to be equalized in the non-distorted economy. See Hopenhayn (2014) for a review.

In the non-distorted economy, without capital wedges, the level of TFP in the Q sector is

$$\mathsf{TFP}^{\star}_{Q} = \left(\int_i (z_i)^{\frac{\mu}{1-\mu}} \, di \right)^{\frac{1-\mu}{\mu}} \equiv \bar{z}. \tag{D.5}$$

Therefore, we can measure the improvement in TFP in the Q sector, Ω_Q, as a result of eliminating the capital wedges, or equivalently, as a result of eliminating the collateral constraints:

$$\Omega_Q \equiv 1 - \frac{\mathsf{TFP}_Q}{\mathsf{TFP}^{\star}_Q} = 1 - \left(\int_i \left(\frac{\bar{z}}{z_i} \frac{\mathsf{TFPR}_{Q,i}}{\overline{\mathsf{TFPR}_Q}} \right)^{\frac{\mu}{1-\mu}} di \right)^{\frac{\mu-1}{\mu}}. \tag{D.6}$$

This measure does not capture the aggregate effect on the economy because (i) it applies only to the Q sector and not to the production of the final good, and (ii) it does not take into account changes in aggregate capital in the efficient economy with respect to the equilibrium of the distorted economy. In our benchmark economy, we obtain a value of $\Omega_Q = 0.35$, implying TFP gains of 35% in the Q sector coming from eliminating the collateral constraints.

Aggregate TFP. The final goods producers operate competitively and face no constraints or distortions, so there is no labor misallocation in the model. Because of this, the only source of misallocation and TFP losses is the Q sector. We can therefore write output as

$$Y = \text{TFP} \cdot K^{\alpha} L^{1-\alpha}, \tag{D.7}$$

where $\text{TFP} \equiv \text{TFP}_Q^{\alpha}$ captures the aggregate TFP of the model. Similarly, we can define the efficient TFP level of the economy as $\text{TFP}^{\star} \equiv (\text{TFP}^{\star})^{\alpha}$ and the aggregate TFP gain from eliminating distortions in the economy as

$$\Omega_Y \equiv 1 - \frac{\mathsf{TFP}}{\mathsf{TFP}^{\star}} = 1 - \left(\frac{\mathsf{TFP}_Q}{\mathsf{TFP}^{\star}_Q} \right)^{\alpha}. \tag{D.8}$$

In our benchmark, the total productivity gain from eliminating the collateral constraints in the Q sector amounts to 16% higher TFP.

Finally, we can use (D.7) to decompose the aggregate effect of tax reforms (say, the revenue-neutral reform) on output into the individual effects on TFP, the level of capital and the level of labor. We can write

$$\frac{Y^{RN}}{Y^{US}} = \frac{\text{TFP}^{RN}}{\text{TFP}^{US}} \left(\frac{K^{RN}}{K^{US}} \right)^{\alpha} \left(\frac{L^{RN}}{L^{US}} \right)^{1-\alpha}. \tag{D.9}$$

See Table E.8 for an application.

E Extensions and Robustness

E.1 Low-Inequality Calibration

Table E.5 – Welfare Change: L-INEQ Calibration

(a) Tax Reform

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	4.0	4.7	5.9	7.7	10.8	12.9	96.3	96.3	98.5	99.2	99.8	99.9
21–34	3.7	3.9	4.5	5.6	7.3	8.2	97.2	96.2	96.7	96.7	95.7	94.1
35–49	2.6	2.4	2.3	2.2	1.7	1.0	92.0	87.9	85.9	82.1	73.7	66.2
50–64	0.8	0.6	0.4	0.1	-0.7	-1.5	66.5	63.0	59.9	54.7	45.7	39.4
65+	-0.7	-0.6	-0.6	-0.8	-1.2	-1.6	2.5	9.9	11.7	11.5	10.6	9.5

(b) Optimal Wealth Taxes

	Distribution of Welfare Gains and Losses						Fraction with Positive Welfare Gain					
	Ability Groups ($\bar{z}_i$ Percentiles)						*Ability Groups ($\bar{z}_i$ Percentiles)*					
	0-40	40-80	80-90	90-99	99-99.9	99.9+	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	5.4	5.7	6.8	8.7	12.1	14.5	94.5	93.2	96.1	97.5	99.0	99.5
21–34	4.9	4.5	4.7	5.5	6.9	7.2	94.9	91.7	92.6	91.9	89.8	86.7
35–49	3.2	2.2	1.7	1.0	-0.5	-2.1	84.7	75.6	72.2	65.1	53.2	43.6
50–64	0.5	-0.2	-0.6	-1.4	-3.1	-4.5	58.1	50.4	45.0	39.0	30.2	24.5
65+	-1.4	-1.5	-1.7	-2.0	-2.9	-3.8	0.6	2.9	3.9	3.8	3.3	2.8

Notes: Each panel reports the average welfare gain (CE_1) and the share of individuals who experience a positive welfare gain (CE_1) in a given age and entrepreneurial productivity group (ranked according to the permanent component of entrepreneurial productivity $\bar{z}$) from the corresponding tax experiment under the low inequality (L-INEQ) calibration. The average and shares are computed with respect to the benchmark distribution of the L-INEQ calibration. All numbers are in percentage points.

Table E.6 – Tax Reform: Change in Macro Variables from Low Inequality Calibration

	Change from the Benchmark of L-INEQ Calibration										
	Quantities (% Change)							*Prices (Change)*			
	K	Q	K/Y	TFP_Q	L	Y	C	$\overline{w}$	$\overline{w}$ (net)	$\Delta r^{\dagger}$	$\Delta r^{\dagger}$ (net)
Revenue-neutral reform	11.2	15.0	3.4	3.4	0.9	6.3	6.4	5.4	5.4	0.0	0.5
Opt. Wealth Taxes	4.0	8.3	4.1	4.1	2.5	4.8	5.9	2.2	7.9	–0.5	–1.1

Notes: RN refers to the revenue-neutral reform and OWT to the optimal wealth tax reform. Percentage changes are computed with respect to the low-inequality calibration, which has $\tau_k = 25\%$ and $\tau_a = 0\%$. †Changes in the interest rate are reported in percentage points. The net wage is defined as $(1-\tau_\ell)\,w$, and the net interest rate is defined as $(1-\tau_k)\,r$ or $r-\tau_a$, depending on the model. The TFP variable is measured in the intermediate goods market.

E.2 Incomplete Markets Model with "Awesome-State" Labor Income Shocks

We consider a version of our benchmark model without return heterogeneity and life-cycle demographics, which essentially becomes a perpetual-youth Aiyagari-style model. We introduce "awesome-state" idiosyncratic income shocks à la Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and try to match their calibration and parameter choices as closely as we can.

In contrast to our benchmark model, there is no individual production of intermediate goods, and all output is produced by the competitive final goods producers that operate a technology

$$Y = K^{\alpha} L^{1-\alpha}, \tag{E.1}$$

where $K \equiv \int a_i di$ is the total amount of capital (or wealth) in the economy. Final good producers rent capital at a rate r and labor at a wage w. In equilibrium it holds that

$$r = \alpha\frac{Y}{K}; \qquad w = (1-\alpha)\,\frac{Y}{L}. \tag{E.2}$$

This production setup is equivalent to the one in our benchmark model when $z_i = \overline{z}\,(=1)$ for all individuals and $\mu = 1$, so there are no monopolistic rents in the production of intermediate goods. All individuals are therefore workers and have a common rate of return r.

We also change the life cycle of individuals to match that in Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Workers are only subject to idiosyncratic labor efficiency shocks. In terms of our benchmark model, only e_i differs across individuals, with no type-dependent (κ_i) or age-dependent variation in labor income. The labor income of an individual is therefore $\overline{w}e_i\ell_i$, where ℓ_i is the endogenously determined labor supply. Workers retire with a constant probability p_{ret}. While retired, workers earn a retirement income ω_{ret} and die with a constant probability p_{death}. Only retirees can die. Upon death, individuals are replaced by a new worker (their descendant) that inherits their assets. In contrast to Castañeda, Díaz-Giménez, and Ríos-Rull (2003), there is no direct correlation between the worker's labor efficiency before retirement and that of their descendant.

We parametrize the model assuming that the labor efficiency shocks follow a discrete Markov process taking n_e values with a transition matrix Π^e. Newborn workers draw their initial labor efficiency from a distribution G^e. We take the number of states ($n_e = 4$) and the transition matrix between the states (Π^e) from Castañeda, Díaz-Giménez, and Ríos-Rull (2003, Table 4). We take the values of e_1, e_2 and e_3 from Castañeda, Díaz-Giménez, and Ríos-Rull (2003, Table 5). The value of e_{n_e} corresponds to the "awesome-state," and we set it to match a share of wealth held by the top 1% of 30%. We set $e_{n_e} = 265$. We take the values for $p_{ret} = 0.022$ and $p_{death} = 0.066$ from Castañeda, Díaz-Giménez, and Ríos-Rull (2003, Table 3). We set the value of ω_{ret} so as to obtain a ratio of transfers to GDP of 4.9% Castañeda, Díaz-Giménez, and Ríos-Rull (2003, pg. 837).

Finally, we set G^e according to *Step 2* of the procedure described in Castañeda, Díaz-Giménez, and Ríos-Rull (2003, Appendix A), overweighting the stationary distribution of labor efficiency γ^e (Castañeda, Díaz-Giménez, and Ríos-Rull, 2003, Table 5) with $\phi_2 = 0.525$ (Castañeda, Díaz-Giménez, and Ríos-Rull, 2003, Table 3) which controls the intergenerational earnings persistence. Accordingly, we set

$$G_1^e = \gamma_1^e + \phi_2\gamma_2^e + \phi_2^2\gamma_3^e + \phi_2^3\gamma_4^e, \tag{E.3}$$

$$G_2^e = (1-\phi_2)\left(\gamma_2^e + \phi_2\gamma_3^e + \phi_2^2\gamma_4^e\right), \tag{E.4}$$

$$G_3^e = (1-\phi_2)\left(\gamma_3^e + \phi_2\gamma_4^e\right), \tag{E.5}$$

$$G_4^e = (1-\phi_2)\,\gamma_4^e. \tag{E.6}$$

To further facilitate comparison, we set the discount factor to $\beta = 0.924$ (Castañeda, Díaz-Giménez, and Ríos-Rull, 2003, Table 3). The remaining parameters of the model are left unchanged with respect to our benchmark.

E.3 Equilibrium with a Corporate Sector

Consider a model with two sectors: corporate and private. The goods of the two sectors are imperfect substitutes and are aggregated into a final good using a Cobb-Douglas technology:

$$Y = Y_c^{\rho} Y_p^{1-\rho}. \tag{E.7}$$

The corporate and private goods are also produced using Cobb-Douglas technologies:

$$Y_c = A K_c^{\alpha} L_c^{1-\alpha}, \qquad Y_p = Q^{\alpha} L_p^{1-\alpha}, \tag{E.8}$$

where $Q = \left(\int x_i^{\mu} di\right)^{1/\mu}$.

The corporate sector firms operate in perfect competition and face no financial constraints. There is a common market for labor with a price $\overline{w}$ per efficiency unit. There is a common capital market for corporate firms and private intermediate goods producers with an interest rate r.

The intermediate private goods x_i are sold by individual monopolists at a price $p(x_i)$ as described in the main text.

Equilibrium Conditions. The first order conditions of the final good aggregator are

$$p_c = \rho \left(\frac{Y_c}{Y_p}\right)^{-(1-\rho)} \qquad p_p = (1-\rho) \left(\frac{Y_c}{Y_p}\right)^{\rho}.$$

From these conditions, we get the following conditions for the expenditure shares across sectors:

$$\frac{p_c Y_c}{Y} = \rho \qquad \frac{p_p Y_p}{Y} = 1 - \rho.$$

The first-order conditions of the corporate sector are

$$r + \delta = p_c \alpha A \left(\frac{K_c}{L_c}\right)^{-(1-\alpha)} \qquad \overline{w} = p_c (1-\alpha) A \left(\frac{K_c}{L_c}\right)^{\alpha}.$$

The first-order conditions of the private sector are

$$p(x_i) = p_p \alpha \left(\frac{x_i}{Q}\right)^{\mu-1} \left(\frac{Q}{L_p}\right)^{-(1-\alpha)} \qquad \overline{w} = p_p (1-\alpha) \left(\frac{Q}{L_p}\right)^{\alpha}.$$

The first-order conditions imply a relationship between corporate and private labor: $L_c/L_p = \rho/1-\rho$. This in turn implies a constant share of labor in the corporate sector of $L_c/L = L_c/L_c+L_p = \rho$.

Calibration. There are only two additional parameters. The share of the corporate sector in the production of the final good, ρ, and the productivity of the corporate sector, A.

Asker, Farre-Mensa, and Ljungqvist (2011) estimate that privately held US firms account for 69% of private sector employment, 59% of aggregate sales, and 53% of aggregate nonresidential fixed investment. In keeping with these estimates, we set $\gamma = 0.4$, which matches the share of the private sector in aggregate sales. Our calibration also implies a share of capital in the private sector of 50%, also in line with the data.

We keep the borrowing limit as in the benchmark, which gives a debt-to-asset ratio of 0.52. This is slightly higher than 0.45 reported by Asker, Farre-Mensa, and Ljungqvist (2011). Finally, we calibrate the remaining parameters to match the same moments as in our baseline in particular, a capital-to-GDP ratio of 3 and a top 1% share of 36%.

Benchmark Outcomes. It is instructive to discuss how the introduction of the corporate sector affects equilibrium outcomes and the calibration. The corporate sector increases the demand for capital, increasing the equilibrium interest rate. The higher interest rate reduces wealth inequality because low productivity entrepreneurs earn higher returns by lending in the bond market, while high productivity entrepreneurs earn lower returns because they face higher borrowing costs. Thus, the calibration produces a higher dispersion in entrepreneurial productivity to match the same top wealth concentration as in the data. These changes imply a slightly higher TFP_Q loss within the private sector (relative to that in our baseline). However, the TFP loss is substantially smaller at the aggregate level. As we illustrate next, the recalibrated model with the corporate sector gives very similar outcomes as our baseline model.

Table E.7 – Robustness: Optimal Wealth Tax

	Baseline		Corporate Sector	
	TR	OWT	TR	OWT
	(1)	(2)	(3)	(4)
τ_a	1.19	3.03	1.24	3.85
τ_ℓ	22.4	15.4	22.4	12.8
τ_k	—	—	—	—
	Change in Welfare (%)			
$\overline{\mathsf{CE}}_1$	6.8	9.0	6.1	9.5
$\overline{\mathsf{CE}}_2$	7.2	8.7	6.3	8.8
	Productivity			
$\frac{\text{TFP}^\star - \text{TFP}}{\text{TFP}^\star}$	0.14	0.13	0.09	0.08
	(Reference - US Economy)			
	0.16		0.11	

Notes: Columns (1) and (2) correspond to the tax reform and optimal wealth tax experiments in the baseline model. Columns (3) and (4) correspond to the tax reform and optimal wealth tax experiments for an extension with a corporate sector that operates a constant returns to scale technology and faces no financial constraints. The outputs of the corporate and private sectors are imperfect substitutes. The "Reference - US Economy" TFP corresponds to the level in the calibrated economy under the baseline US tax system, with $\tau_k = 0.25$ and $\tau_a = 0$.

Tax Reform and Optimal Wealth Taxes. The mechanisms that operate in our baseline model are also present in the extension with a corporate sector. An increase in wealth taxes favors the accumulation of capital by high-return individuals, who are themselves entrepreneurs producing in the private sector. Because of this reallocation of capital, productivity in the private sector improves, resulting in higher overall productivity and an increase in output in the private sector relative to that of the corporate sector. The outcome of the tax reform and optimal tax experiments is similar to those in the baseline model, in terms of both the level of taxes and the magnitude of the welfare gains. See Table E.7.

The optimal wealth tax is 3.8% and implies a welfare gain of 8.8% for newborn agents. The gains are carried by improvements in productivity that raise wages. Total factor productivity increases so that the distance from the efficient productivity (i.e., $(\text{TFP}^\star - \text{TFP})/\text{TFP}^\star$) falls from 0.11 to 0.08 (compared with a fall from 0.16 to 0.13 in our baseline model).

This rise is explained by the reallocation of capital among private businesses and between the corporate and the private sector. TFP_Q increases by 14%, while the share of capital in the corporate sector decreases from 52% to 51%. This reallocation of capital happens as total capital decreases by 2.5%. The decrease is higher in the corporate sector (3.6%) than in the private sector (1.3%). Despite the decrease in the level of capital, both corporate and private output increase, by 1% and 7% respectively, resulting in a 4.7% increase in total output.

How to Compute TFP? The aggregate production function can be written as

$$Y = Z\hat{K}^{\alpha}\hat{L}^{1-\alpha}, \tag{E.9}$$

where $\hat{K} \equiv K_c^{\rho} K_p^{1-\rho}$ and $\hat{L} \equiv L_c^{\rho} L_p^{1-\rho}$ are aggregated inputs. Total factor productivity is

$$Z \equiv \frac{Y}{\hat{K}^{\alpha}\hat{L}^{1-\alpha}} = A^{\rho}\left(\text{TFP}_Q\right)^{\alpha(1-\rho)}. \tag{E.10}$$

To decompose the change in total output between two allocations, Y and Y', write in logs

$$\log\left({}^{Y'}\!/_{Y}\right) = \underbrace{\alpha\log\left(\frac{K'}{K}\right)}_{\text{Total Capital}} + \underbrace{(1-\alpha)\log\left(\frac{L'}{L}\right)}_{\text{Total Labor}} + \underbrace{\log\left(\frac{Z'}{Z}\right)}_{\text{Productivity}} \tag{E.11}$$

$$+\,\alpha\underbrace{\left(\rho\log\left(\frac{K_c'/K'}{K_c/K}\right) + (1-\rho)\log\left(\frac{K_p'/K'}{K_p/K}\right)\right)}_{\text{Realocation of Capital Across Sectors}}.$$

We implement this decomposition in Table E.8. Most of the change in output comes from increases in productivity, carried by reallocation within the private sector.

E.4 Extension with Public Firms

We consider another extension in which firms stochastically become "public," by which we mean they face a substantial increase in their access to credit. In this version, entrepreneurial productivity is heterogeneous but fixed (i.e., $z_{ih} = \bar{z}_i$ for all i and h, unless $\mathbb{I}_{ih} = 0$ so $z_{ih} = 0$), but each period, a firm exogenously transitions to become public with probability p_{public} and sees a jump in its collateral ratio to $\bar{\vartheta}_{\text{public}} \gg \bar{\vartheta}(z)$. Public and private firms also differ in the probability with which they exit ($\mathbb{I}_{ih} = 0$), with private firms exiting at a higher rate.

Entrepreneurial Ability and Productivity. The entrepreneurial *productivity* of individual i at age h, denoted z_{ih}, has two components: their entrepreneurial *ability*, $\bar{z}_i$, which is a fixed characteristic of the individual, and a second component that determines whether the individual's firm is active and, if so, whether it operates as a "private" or "public" firm. The ability component is transmitted imperfectly from a parent to her child just as in the benchmark model:

$$\log\left(\bar{z}_i^{\text{child}}\right) = \rho_z \log\left(\bar{z}_i^{\text{parent}}\right) + \varepsilon_{\bar{z}_i}, \tag{E.12}$$

where $\varepsilon_{\bar{z}_i} \sim \mathcal{N}\left(0, \sigma^2_{\bar{z}_i}\right)$.

Table E.8 – Decomposition of the Change in Output

		Contributions of				
		ΔK	ΔL	ΔTFP	Reallocation of capital	
$\log\left(\frac{Y'}{Y}\right) =$		$\alpha\log\left(\frac{K'}{K}\right)$	$(1-\alpha)\log\left(\frac{L'}{L}\right)$	$\log\left(\frac{Z'}{Z}\right)$	$\alpha\rho\log\left(\frac{K_c'/K'}{K_c/K}\right)$	$\alpha(1-\rho)\log\left(\frac{K_p'/K'}{K_p/K}\right)$
Baseline						
Tax Reform	8.84	6.08	0.7	2.06	—	—
OWT	5.94	1.04	1.94	2.97	—	—
Corp. Model						
Tax Reform	8.43	5.63	0.79	1.94	–0.14	0.21
OWT	4.60	–1.01	2.38	3.12	–0.19	0.29

Notes: The contribution of TFP is computed from the change of TFP_Q, and it corresponds to $\alpha(1-\rho)\log\left(\text{TFP}_Q'/\text{TFP}_Q\right)$, with $\rho = 0$ in the baseline model. There is no reallocation of capital across sectors in the baseline model, because all output is produced by the private sector.

There are three states for the firm $\mathbb{I}_{ih} \in \{\mathcal{P}r, \mathcal{P}u, 0\}$, corresponding to private, public, and inactive, respectively. Private and public firms operate with a productivity equal to the owner's entrepreneurial ability ($z_{ih} = \bar{z}_i$), while inactive firms have no productivity ($z_{ih} = 0$) and hence do not operate. The private and public status of firms is inherited across generations, capturing firms being inherited upon death of the previous owners. New firms are all private, so that if an individual with $\mathbb{I}_{ih} = 0$ dies, their offspring will operate a private firm.

High productivity private firms (those with $\bar{z}_i > \bar{z}_{\text{median}}$) have a probability p_{public} of becoming public, and active firms have a probability $p_0^{\mathcal{P}r}$ and $p_0^{\mathcal{P}u}$ of becoming inactive, which depends on their private/public status. Inactive firms remain so. The evolution of z_{ih} can be summarized by the following three-state Markov chain:

$$z_{ih} = \begin{cases} \bar{z}_i & \text{if } \mathbb{I}_{ih} = \mathcal{P}r \\ \bar{z}_i & \text{if } \mathbb{I}_{ih} = \mathcal{P}u \\ 0 & \text{if } \mathbb{I}_{ih} = 0 \end{cases} \quad \text{and} \quad \Pi_{\mathbb{I}} = \begin{bmatrix} 1 - p_{\text{public}} - p_0^{\mathcal{P}u} & p_{\text{public}} & p_0^{\mathcal{P}r} \\ 0 & 1 - p_0^{\mathcal{P}u} & p_0^{\mathcal{P}u} \\ 0 & 0 & 1 \end{bmatrix}. \tag{E.13}$$

Financial Markets. There is a bond market in which intra-period borrowing and lending take place at interest rate, r. The market works in the same way as in our benchmark. The access to the market depends on the entrepreneur's ability and the private/public status of the firm. Borrowing is collateralized and is subject to a limit indexed to individuals' assets:

$$k_{ih} \leqslant \vartheta(\bar{z}_i, \mathbb{I}_{ih}) \times a_{ih}. \tag{E.14}$$

For private firms, we keep the same properties as in the benchmark, with $\vartheta(\bar{z}_i, \mathcal{P}r) \geqslant 1$ and $\vartheta'(\bar{z}_i, \mathcal{P}r) > 0$. Public firms have more access to credit and so $\vartheta(\bar{z}_i, \mathcal{P}r) = \bar{\vartheta}_{\text{public}} \gg \max\vartheta(\bar{z}_i, \mathcal{P}r)$.

TABLE E.9 – Robustness: Optimal Wealth Tax

	Tax Reform		OWT	
	Baseline	Public Firms	Baseline	Public Firms
τ_a	1.13	1.52	3.03	2.76
τ_ℓ	22.4		15.4	17.6
	Change in Welfare (%)			
$\overline{\mathsf{CE}}_1$	6.8	4.4	9.0	5.9
$\overline{\mathsf{CE}}_2$	7.2	4.1	8.7	4.8

Notes: The table reports taxes, welfare gain for the revenue-neutral tax reform and optimal wealth tax economies for the benchmark model and the alternative model with firms with increased credit access. All numbers are in percentage points.

Parameterization. We set all parameters as in the benchmark, with the exception of the discount factor β, the consumption share in utility γ, the strength of the bequest motive χ, the dispersion of the labor fixed effect $(\sigma_{\varepsilon_\kappa})$ and of the entrepreneurial ability $(\sigma_{\varepsilon_{\bar{z}}})$, and the new parameters $\left\{p_0^{\mathcal{P}r}, p_0^{\mathcal{P}u}, p_{\text{public}}, \overline{\vartheta}_{\text{public}}\right\}$. We set these parameters to jointly match a capital-to-output ratio of 3.0, an average number of labor hours of 0.4, a bequest-to-wealth ratio of 1.2 percent, a standard deviation of log earnings of 0.8, and a top 1% wealth share of 37% as in the benchmark. We also target a share of public firms of 0.5% and a leverage ratio of 90% for public firms.[54]

The calibration implies high levels of debt, with a debt-to-output ratio of 2.43, carried by public firms that account for 88% of debt. We keep the borrowing of private firms as in the benchmark, with the lowest-ability group, $\bar{z}_0$, not being able to borrow at all $(\vartheta(\bar{z}_0, \mathcal{P}r) = 1)$, and the borrowing limit increasing linearly with ability from there on: $\vartheta(\bar{z}) = 1 + \varphi(\bar{z} - \bar{z}_0)$ with $\varphi = 0.225$. Wealth concentration is also higher than in the benchmark, with a top 0.1% wealth share of 28%.

Tax Reform and Optimal Wealth Tax. We conduct the same tax reform and optimal wealth tax experiments as we did in our benchmark. The substantive results in terms of efficiency and welfare gains from replacing capital income with wealth taxes remain unchanged; however, the size of the gains in both TFP and welfare are lower. The TFP gains are about one-half of what they are in our benchmark and welfare gains are between one-half and two-thirds, depending on the welfare measure. The general pattern across aggregates is the same as before. The level of the optimal wealth tax is lower (2.76%).

[54] We target the ratio of public firms in the US from Compustat relative to the number of firms with at least five employees from the Business Dynamics Statistics of the US Census Bureau.

E.5 Additional Robustness and Extensions

Table E.10 reports the results of nine additional robustness experiments to complement those reported in Table XII: (i) calibrating to looser constraints by targeting a debt-to-GDP ratio of 2.5, (ii) making borrowing constraints independent of productivity, $\vartheta(z) = \vartheta$; (iii) reducing the CES curvature to $\mu = 0.8$; (iv) removing life-cycle stochastic variation in productivity, $z_i = \bar{z}_i$; (v) having all individuals be born in the middle lane and transition to fast lane with probability $p_3 = 3\%$; and (vi-ix) adding wealth taxes *on top of* the current tax system with revenues used for wasteful spending or rebated by reducing the labor income tax.

TABLE E.10 – Additional Robustness and Extensions: Optimal Wealth Tax

	Looser Constraints	Constant ϑ	Higher Markups	Constant Productivity	No Start in Fast Lane	Add τ_a to Benchmark			
						2% Wealth Tax		OWT Wealth Tax	
	debt/GDP = 2.5	$\vartheta(z) = \overline{\vartheta}$	$\mu = 0.8$	$z_{ih} = \overline{z}_i$	$z_{ih} = \overline{z}_i$	τ_ℓ fixed	Adjust τ_ℓ	τ_ℓ fixed	Adjust τ_ℓ
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
τ_a	2.34	3.66	2.45	2.16	2.8	2.00		3.03	
τ_ℓ	19.5	12.4	18.0	19.4	16.1	22.4	14.9	22.4	12.0
					Welfare Change				
$\overline{CE}_1$	4.4	11.8	8.2	6.0	8.5	–8.3	0.9	–11.9	0.3
$\overline{CE}_2$	4.2	11.2	7.6	5.5	8.2	–9.9	0.0	–14.2	–1.0
					Change in Macro Variables (%)				
K	4.0	0.8	6.5	–2.3	2.8	–20.5	–15.6	–28.7	–22.6
Q	6.6	12.0	12.4	17.0	11.8	–17.5	–13.0	–24.7	–19.1
Y	3.6	7.3	6.1	8.0	6.5	–6.5	–4.0	–9.5	–6.2
L	1.6	4.3	2.0	2.4	3.2	1.6	2.5	2.3	3.5
C	4.1	10.1	7.3	11.1	8.5	–9.6	–2.5	–13.6	–4.2
TFP_Q	2.5	11.1	5.6	19.7	8.8	3.8	3.0	5.6	4.6
$\overline{w}$	2.0	2.9	3.9	5.5	3.3	–8.0	–6.4	–11.5	–9.4
$\overline{w}$ (net)	5.7	16.1	9.9	9.6	11.6	–8.0	2.7	–11.5	2.8
					Benchmark Economy's Debt and Productivity ($\tau_k = 0.25$, $\tau_a = 0.0$, $\tau_\ell = 0.224$)				
debt/GDP	2.5	1.5	1.5	1.5	1.5	1.5			
$\frac{TFP^\star - TFP}{TFP^\star}$	0.05	0.21	0.14	0.21	—	0.16			

Notes: The nine additional robustness experiments are as follows: (i) calibrating to looser constraints by targeting debt/GDP ratio of 2.5, (ii) making borrowing constraints independent of productivity, $\vartheta(z) = \vartheta$; (iii) reducing the CES curvature to $\mu = 0.8$; (iv) removing lifecycle stochastic variation in productivity, $z_i = \overline{z}_i$; (v) having all individuals born in the middle lane and transition to fast lane with probability $p_3 = 3\%$; (vi-ix) adding wealth taxes on top of the current tax system with revenues used for wasteful spending or rebated by reducing the labor income tax.

TABLE E.11 – Robustness Additional Results: Optimal Wealth Tax

	Baseline	Credit Spread		Public	Corporate	Pure Rents	Non-linear OKIT	
	OWT	10.1%	6%	Firms	Sector	Model	$\tilde{\tau}_k(y) = y - \psi y^{\eta}$	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
τ_a	3.03	2.33	2.46	2.76	3.25	1.40	—	—
τ_ℓ	15.4	13.6	15.5	17.6	16.3	27.0	22.4 (fixed)	32.3
τ_k	—	—	—	—	—	—	$\underset{(\psi,\eta)}{(0.73, 1.022)}$	$\underset{(\psi,\eta)}{(1.20, 0.992)}$
				Change in Welfare (%)				
$\overline{CE}_1$	9.0	6.1	4.3	5.9	5.8	–1.7	0.9	4.2
$\overline{CE}_2$	8.7	5.6	3.5	4.8	5.5	–1.4	0.8	5.4
				Change in Macro Variables (%)				
K	2.6	4.4	–0.8	1.5	–2.1	–2.5	5.4	41.3
Q	10.5	2.9	–1.1	4.8	11.0	–2.4	6.3	49.2
Y	6.1	3.0	0.9	3.3	2.5	–2.4	3.1	16.2
L	1.2	3.0	2.4	2.2	2.5	–2.4	1.0	–1.6
C	9.5	5.3	3.0	4.3	3.5	–3.0	3.1	14.3
TFP_Q	7.7	–1.5	–0.3	3.2	7.7	0.1	0.8	5.6
$\overline{w}$	2.8	–0.1	–1.4	1.0	0.0	–0.0	2.1	18.1
$\overline{w}$ (net)	12.0	11.3	7.3	7.3	7.9	–5.9	2.1	3.0
				Benchmark Economy's Debt and Productivity ($\tau_k = 0.25$, $\tau_a = 0.0$, $\tau_\ell = 0.224$)				
debt/GDP	1.5	1.5	2.0	2.4	0.76	1.1	1.5	
$\frac{\text{TFP}^{\star}-\text{TFP}}{\text{TFP}^{\star}}$	0.16	0.077	0.027	0.14	0.065	0.004	0.16	

Notes: The seven robustness experiments are as follows: (1) replacing collateral constraints with unlimited borrowing, subject to a credit spread of 10% generating a debt-to-GDP ratio of 1.5; (2) same as (1) but with a spread of 6%; (3) allowing firms to stochastically transition to relaxed collateral constraints; (4) introducing a corporate sector with Cobb-Douglas production and no borrowing limits; (5) eliminating z_i heterogeneity to focus on pure monopolistic rents; (6) tax reform that replaces τ_k with a nonlinear capital income tax; and (7) optimal nonlinear capital income tax experiment (choosing ψ, η, τ_ℓ).

F Endogenous Entrepreneurial Hours

In the baseline formulation, entrepreneurs' labor supply does not enter their production function. This was a deliberate choice to avoid introducing another (potentially interesting) channel through which wealth and capital income taxes can operate, which would add another layer to the analysis. Leaving a full analysis to future research, we show in this section how a plausible extension that introduces labor supply would interact with wealth taxes. The main result is that the labor supply of entrepreneurs would rise under wealth taxes, relative to the supply under capital income taxes, as long as their initial labor hours are not too high, and vice versa when they are. We give a sketch of this result here and provide more details and derivations in the following subsection.

F.1 Overview of Result

The main new channel results from a standard income versus substitution effect. To see this, consider the modified production function, $x = z(k\ell)^{\mu}$, replacing (8), so the entrepreneurs' problem (16) becomes

$$\max_{\ell, k \leqslant \vartheta(z) a} \left((1-\tau_a)a + [R(zk\ell)^{\mu} - (r+\delta)k + ra](1-\tau) - a'\right)^{\gamma}(1-\ell)^{1-\gamma},$$

where $\tau \in \{\tau_a, \tau_k\}$ and $\tau_a = 0$ if $\tau_k > 0$. The first order condition for hours is given as

$$(1-\tau)\mu R(zk)^{\mu}\ell^{\mu-1}(1-\ell) = \frac{1-\gamma}{\gamma}\left((1-\tau_a)a + [R(zk\ell)^{\mu} - (r+\delta)k + ra](1-\tau) - a'\right).$$

The left-hand side corresponds to the marginal benefit of extra work, which is the marginal utility of consuming extra output. The marginal utility depends on leisure, since consumption and leisure are complements in the utility function. So, when ℓ is high, that is, when leisure is low, the marginal benefit (MB) of extra work is lower. Switching to a wealth tax increases MB, because τ_a is a much smaller tax than τ_k on output. But if ℓ is high, the increase in MB will be small. Now, consider the marginal cost (MC): it is the utility loss due to extra work, which is proportional to consumption due to complementarity. If a switch to a wealth tax reduces consumption, it is obvious that ℓ increases. But if the wealth tax raises her consumption, what happens to ℓ depends on how much MB increases relative to MC. We can show that for our benchmark parameterization, a sufficient condition for hours to increase is $\ell \leqslant 0.43$ for the capital-constrained entrepreneur and $\ell \leqslant 0.88$ for the unconstrained entrepreneur.

To see this, consider the problem of an entrepreneur who chooses hours of work ℓ in her own firm and capital:

$$\max_{\ell, k \leqslant \vartheta(z) a} \left((1-\tau_a)a + [R(zk\ell)^{\mu} - (r+\delta)k + ra](1-\tau) - a'\right)^{\gamma}(1-\ell)^{1-\gamma},$$

where $\tau \in \{\tau_a, \tau_k\}$ and $\tau_a = 0$ if $\tau_k > 0$. The first-order condition with respect to ℓ gives

$$\frac{dC}{d\ell}C^{\gamma-1}(1-\ell)^{1-\gamma} \quad = \quad \left(\frac{1-\gamma}{\gamma}\right)C^{\gamma}(1-\ell)^{-\gamma}.$$

The left-hand side is the marginal benefit, and the right-hand side is the marginal cost of extra hours of work in one's firm. Simplifying this expression and substituting consumption gives

$$(1-\tau)\mu R(zk)^{\mu}\ell^{\mu-1}(1-\ell) = \tfrac{1-\gamma}{\gamma}\left((1-\tau_a)a + [R(zk\ell)^{\mu} - (r+\delta)k + ra](1-\tau) - a'\right).$$

F.2 Details and Derivations

A. Capital-Constrained Entrepreneur $(k = \vartheta(z)a)$

In this case, $k = \vartheta(z)a$ is fixed, and the first order condition is given by the following:

$$(1-\tau)\,\mu R\,(z\vartheta\,(z)\,a\ell)^{\mu}\,\tfrac{1-\ell}{\ell}=$$
$$\tfrac{1-\gamma}{\gamma}\,((1-\tau_a)\,a+[R\,(z\vartheta\,(z)\,a\ell)^{\mu}-(r+\delta)\,\vartheta\,(z)\,a+ra]\,(1-\tau)-a')\,.$$

The left-hand side decreases with ℓ and the right-hand side increases with ℓ; thus, there is a unique solution. Consider what happens to the left-hand side and right-hand side for a given ℓ if we switch from a capital income tax to a wealth tax:

$$\Delta\mathsf{LHS}=(\tau_k-\tau_a)\,R\,(z\vartheta\,(z)\,a\ell)^{\mu}\,\mu\frac{1-\ell}{\ell}$$
$$\Delta\mathsf{RHS}=\frac{1-\gamma}{\gamma}\left(-\tau_a a+(\tau_k-\tau_a)\,[R\,(z\vartheta\,(z)\,a\ell)^{\mu}-(r+\delta)\,\vartheta\,(z)\,a+ra]-\Delta a'\right).$$

If $\Delta\mathsf{LHS}>\Delta\mathsf{RHS}$, then ℓ would increase. To see the conditions under which this would happen, note that the same term $(\tau_k-\tau_a)\,R\,(z\vartheta\,(z)\,a\ell)^{\mu}$ appears on both sides. However, there are some additional negative terms on the right-hand side:

1. $-(r+\delta)\,\vartheta\,(z)\,a+ra<0$,
2. $-\Delta a'<0$ if $\Delta C>0$ (the case where $\Delta C<0$ obviously gives an increase in ℓ), and
3. $-\tau_a a<0$.

So, definitely $(\tau_k-\tau_a)\,R\,(z\vartheta\,(z)\,a\ell)^{\mu}>\Delta C$. Thus, if $\mu\frac{1-\ell}{\ell}>\frac{1-\gamma}{\gamma}$, we definitely know that $\Delta\mathsf{LHS}>\Delta\mathsf{RHS}$. Using our benchmark parameterization $\mu=0.9$ and $\gamma=0.46$, we have

$$\begin{aligned}\frac{1-\ell}{\ell}&\geqslant 1.3\\ \frac{1}{\ell}&\geqslant 2.3\\ \ell&\leqslant 0.43.\end{aligned}$$

Of course, this is a sufficient condition. So, if the entrepreneur were not working too much initially (i.e. $\ell\leqslant 0.43$), then switching to a wealth tax would increase her entrepreneurial hours. Otherwise, the income effect would be greater than the substitution effect, and she would reduce her entrepreneurial hours. If we used $\mu=0.45$ and $\gamma=0.46$ instead, the entrepreneurial hours would increase if

$$\ell\leqslant 0.28.$$

B. Capital-Unconstrained Entrepreneur

When the entrepreneur is not capital constrained, we have the same first-order condition for labor supply:

$$(1-\tau)\,\mu R\,(zk)^{\mu}\,\ell^{\mu-1}\,(1-\ell)=\tfrac{1-\gamma}{\gamma}\,((1-\tau_a)\,a+[R\,(zk\ell)^{\mu}-(r+\delta)\,k+ra]\,(1-\tau)-a')\,.$$

The first-order condition for k is given as

$$\begin{aligned}\mu k^{\mu-1} R (z\ell)^{\mu} &= r+\delta \\ k &= \left(\frac{\mu R (z\ell)^{\mu}}{r+\delta}\right)^{1/(1-\mu)}.\end{aligned}$$

Inserting the latter into consumption, we obtain

$$C = (1-\tau_a) a + \left[\left(\frac{\mu R z^{\mu}}{r+\delta}\right)^{1/(1-\mu)} \ell^{\mu/(1-\mu)} (r+\delta) \frac{1-\mu}{\mu} + ra\right](1-\tau) - a',$$

and inserting it into $\mu R (zk)^{\mu} \ell^{\mu-1}$ on the left-hand side of the first-order condition for labor supply gives

$$\begin{aligned}\mu R (zk)^{\mu} \ell^{\mu-1} &= \mu R z^{\mu} \ell^{\mu-1} \left(\frac{\mu R (z\ell)^{\mu}}{r+\delta}\right)^{\mu/(1-\mu)} \\ &= \left(\frac{\mu R z^{\mu}}{(r+\delta)^{\mu}}\right)^{1/(1-\mu)} \ell^{(2\mu-1)/(1-\mu)}.\end{aligned}$$

Using the expression for C and $\mu R (zk)^{\mu} \ell^{\mu-1}$, we can write the first-order condition for labor supply as

$$\begin{aligned}(1-\tau)\left(\frac{\mu R z^{\mu}}{(r+\delta)^{\mu}}\right)^{1/(1-\mu)} \ell^{(2\mu-1)/(1-\mu)} (1-\ell) &= \\ \frac{1-\gamma}{\gamma}\left((1-\tau_a) a + \left[\left(\frac{\mu R z^{\mu}}{(r+\delta)^{\mu}}\right)^{1/(1-\mu)} \ell^{\mu/(1-\mu)} \frac{1-\mu}{\mu} + ra\right](1-\tau) - a'\right).\end{aligned}$$

The left-hand side of this equation corresponds to the marginal benefit, and the right-hand side corresponds to the marginal cost of extra hours of work by the entrepreneur. A switch to a wealth tax increases the left-hand side (since $\tau_a << \tau_k$). At an interior ℓ, that will increase hours of work. The right-hand side might increase or decrease with such a switch. If it decreases, then optimal hours of work increase unambiguously. For example, for wealth-rich entrepreneurs with relatively modest productivity, a wealth tax might reduce their after-tax wealth and consumption, leading them to work more.[55] Consider what happens to the left-hand and the right-hand sides

[55]When $\mu < 0.5$, the left-hand side is strictly decreasing and the right-hand side is strictly increasing and strictly concave in ℓ. Thus, the increase the left-hand side increases hours of work, and the increase in right-hand side reduces hours of work. When $\mu > 0.5$, the right-hand side would be strictly increasing and convex in ℓ. The left-hand side is strictly concave and has a maximum at $\ell = \frac{2\mu-1}{\mu}$. To see this, take the derivative of the left-hand side to obtain

$$\frac{d\mathsf{LHS}}{d\ell} = \text{a } (+) \text{ constant} \times \ell^{\left(\frac{2\mu-1}{1-\mu}\right)} \times \frac{2\mu-1-\mu\ell}{(1-\mu)\ell}.$$

Note that $\mathsf{LHS} = 0$ and $\mathsf{RHS} > 0$ for $\ell = 0$, so the net benefit (MB-MC) of extra hours of work at $\ell = 0$

for a given ℓ if we switch from a capital income tax to a wealth tax:

$$\Delta \mathsf{LHS} = (\tau_k - \tau_a)\left(\frac{\mu R z^{\mu}}{(r+\delta)^{\mu}}\right)^{1/(1-\mu)} \ell^{(2\mu-1)/(1-\mu)}(1-\ell)$$

and

$$\Delta \mathsf{RHS} = \frac{1-\gamma}{\gamma}\left(\tau_k r a - \tau_a (1+r) a + (\tau_k - \tau_a)\left(\frac{\mu R z^{\mu}}{(r+\delta)^{\mu}}\right)^{1/(1-\mu)} \frac{(1-\mu)\ell^{\mu/(1-\mu)}}{\mu} - \Delta a'\right).$$

Note that if the $\Delta\mathsf{RHS} < 0$, the switch to a wealth tax definitely increases entrepreneurial hours. So, we will focus on the case in which $\Delta\mathsf{RHS} > 0$. In this case, $\Delta a' > 0$ because of monotonicity. We also know from all our experiments that a wealth tax puts a higher tax burden on the majority of the population and those who earn the market interest rate. So, we will work with the assumption that $\tau_k r a - \tau_a (1+r) a < 0$. Then, a sufficient condition for $\Delta\mathsf{LHS} > \Delta\mathsf{RHS}$ is that

$$\ell^{(2\mu-1)/(1-\mu)}(1-\ell) \geqslant \frac{1-\gamma}{\gamma}\frac{(1-\mu)\ell^{\mu/(1-\mu)}}{\mu},$$

which implies

$$\begin{aligned}
\frac{1}{\ell} &\geqslant \frac{1-\gamma}{\gamma}\frac{1-\mu}{\mu}+1 \\
\frac{1}{\ell} &\geqslant \frac{(1-\gamma)(1-\mu)+\gamma\mu}{\gamma\mu} \\
\ell &\leqslant \frac{\gamma\mu}{(1-\gamma)(1-\mu)+\gamma\mu}.
\end{aligned}$$

In our calibration, $\gamma = 0.46$ and $\mu = 0.9$, which gives $\ell < 0.88$. If we set $\mu = 0.45$, then $\ell < 0.41$.

is negative. If there is an optimal interior $\ell^* > 0$, then the left-hand side should be above the right-hand side for $\ell < \ell^*$, and the slope of the left-hand side should be smaller than the slope of the right-hand side at $\ell = \ell^*$. Thus, again the increase in the left-hand side increases hours of work, and the increase in the right-hand side reduces hours of work.