# Skewed Business Cycles*

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**Abstract**

Using establishment and firm panel data from the US Census and almost fifty other countries, we show that the skewness of the growth rates of employment, sales, and productivity is procyclical. Firm growth rates display a large negative left tail during recessions and a large positive right tail during expansions. We find similar results at the industry level: industries that are contracting see more left-skewed growth rates of firm sales, employment, and productivity, and industries that are expanding see more right-skewed growth rates. These firm-level negative skewness shocks foreshadow a significant decline in macroeconomic activity, even after controlling for the first and second moments of firm growth. We explain these findings with a heterogeneous-agent model in which risk-averse entrepreneurs face a time-varying distribution of productivity shocks, leading them to cut back on hiring and investment when facing greater downside risk. This suggests the fluctuating risk of large negative and positive firm-level shocks can be an important driving force of business cycles.

**JEL Codes:** E3
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# 1 Introduction

This paper studies how the distribution of firm-level shocks varies over the business cycle. In the previous literature, recessions have usually been characterized as a combination of a negative first-moment (mean) shock and/or a positive second-moment (uncertainty) shock. In this paper, we argue that recessions are also accompanied by negative third-moment (skewness) shocks implying that, during economic downturns, a subset of firms does extremely poorly, leading to a left tail of large negative outcomes. At the same, time another subset of firms experiences muted growth rates, compressing the right tail of (positive) outcomes. In this sense, negative skewness captures a combination of elevated "downside risk" and muted "upside surprises." For example, although major disruptions—such as the 9/11 attacks, the Great Recession, or the COVID-19 pandemic—impact all firms, a subset in certain industries (e.g., airlines, automotive or movie theaters respectively) fared much worse. Hence, recessions can be viewed as periods of heightened occurrence of *firm-level disasters*. This is often accompanied by a deceleration of growth for firms at the top end, leading to a compression of the right tail of positive outcomes. The opposite patterns happen during expansions, with the left tail of firm growth shrinking and the right tail expanding, indicating an increase in rapid growth potential. Consequently, the skewness of firms' growth rates is procyclical.

Using firm-level panel data from the US Census Bureau, Compustat, and panel data on firms from 46 other countries, we show that the cross-sectional skewness of several firm-level outcomes, such as sales growth, employment growth, stock returns, and productivity, is strongly procyclical. As an illustration of our main empirical result, the top panel of Figure 1 displays the distribution of firms' sales growth from the US Census' Longitudinal Business Database (LBD). The solid line shows the empirical density of firms' sales growth from the most recent pre-COVID recessions, 2001–02 and 2008–09. The dashed line instead shows the density for the expansion years around these recessions, years 2003–06 and 2010–14. The distribution of growth during recessions has a thicker left tail than during expansions, indicating an increase in dispersion that is mostly coming from the left tail.

This asymmetric change in the distribution of sales growth from expansion to recession years can be quantified using the Kelley skewness (Kelley, 1947), a measure of skewness that is robust to outliers.[1] The Kelley skewness of sales growth is 0.10 in expansion years (a positively skewed distribution) and –0.11 in recession years (a negatively

[1]Kelley skewness is the difference between the 90th-to-50th percentile differential (a measure of dispersion in the right tail) and the 50th-to-10th percentile differential (a measure of dispersion in

skewed distribution). This value of 0.10 for skewness during expansions indicates that 45% of the overall dispersion is accounted for by firms with sales growth below the median, whereas during recessions, a Kelley value of –0.11 indicates that this share increases to about 55%. The bottom panel of Figure 1 displays the distribution of employment growth from LBD firms for the same expansion and recession years, again highlighting how recessions have a widening left tail and negative skewness.[2]

A second illustration of our results comes from the COVID recession. The top panel of Figure 2 shows different percentiles of the one-year *expected* sales growth distribution for a representative sample of around 500 firms in the Federal Reserve Bank of Atlanta's survey panel. It is clear from this picture that the pandemic generated an increase in the probability of large drops in sales.[3] Pre-pandemic, firms' 10th percentile expected growth predicted a 1% decline in sales, but this dropped to a –15% change at the outset of the pandemic in April 2020. In contrast, the right tail of the probability distribution of expected growth was almost unchanged relative to pre-pandemic periods. As a consequence, the Kelley skewness of the distribution declined from an average of 0.24 before the pandemic to –0.17 during the first months of the recession. Interestingly, the median of the distribution shows much smaller changes, going from 5% to 0%, indicating how the pandemic was mainly associated with a large drop in the left tail of expected sales growth. The bottom panel of Figure 2 shows the monthly realized one-year sales growth distribution, displaying a similar pattern. In this case, the 10th percentile of the distribution drops by about 35%, and the Kelley skewness of *realized* sales growth falls from 0.1 pre-pandemic to –0.35. This evidence suggests that the COVID recession was characterized by a left-skewed distribution of expected *and* realized sales growth. On the flip side, as the US economy emerged from the recession in 2021, the right tail expanded and the left tail shrank, resulting in right skewness in the distribution during the expansion.[4]

the left tail) divided by the difference between the 90th and 10th percentiles (a measure of the total dispersion of the distribution). For a distribution with a compressed upper half and a dispersed lower half, the Kelley skewness is negative (i.e., a left-skewed distribution).

[2]Appendix Figure A.1 plots the arc-percent growth measure from –2 to +2 showing similarly procyclical skewness. Figure A.2 shows similar results for Compustat firms.

[3]Monthly data for the United States from 2017 onwards is obtained from the Survey of Business Uncertainty described in Altig *et al.* (2020) and Barrero (2022). Additional details are in Appendix B.1, including Figure A.3 showing that the monthly Kelley skewness for realized and expected sales growth is procyclical.

[4]These patterns can be also appreciated in the top panel of Figure A.3, which displays the Kelley skewness of expected and realized sales growth for the same sample of firms. The skewness of expected sales growth drops quite rapidly from 0.38 in February 2020 to –0.35 in April 2020, the month when the unemployment rate reached its peak at 14.7%. We find a similar pattern in the realized sales growth

We find the same empirical pattern at the four-digit NAICS industry level in the United States: the *within-industry* skewness of firm-level employment growth, sales growth, productivity growth, and stock returns are strongly positively correlated with industry average growth rates. Furthermore, using firm-level data for 47 countries that are both geographically and economically diverse, we show a similar pattern globally, with the skewness of firm-level outcomes robustly procyclical.

Given these empirical patterns, we then evaluate to what extent fluctuations in firm skewness can account for variations in aggregate output, employment, consumption, and investment. To this end, we use two empirical approaches. First, we run a series of local projection regressions (Jordà, 2005) that show that a shock to the skewness of the distribution of firms' stock returns forecasts an economically and statistically significant decline in aggregate economic activity. In particular, we find that a one-standard-deviation shock to the skewness of stock returns, foreshadows a 0.5% decline in quarterly GDP after four quarters, a 0.3% decline in employment and consumption, and a 2% decline in quarterly investment. Interestingly, these results are symmetrically driven both by the negative growth after negative skewness shocks and positive growth after positive skewness shocks. Second, we exploit cross country-industry variation in the skewness of firms' TFP shocks to show that firms in industries experiencing a decline in the skewness of TFP shocks also experience a significant drop in sales, employment, and investment. Quantitatively, we find that a drop in the within-industry Kelley skewness of firms' TFP shocks from 0.10 to 0 is followed by a decline of 2.9% in sales, 1.3% in employment, and 0.8% in capital investment.

One immediate question is about causality. Is negatively skewed firm growth an endogenous outcome of a negative macro shock, or an exogenous event that helps to drive business cycles? We provide evidence below that these skewness shocks are, at least partly, exogenous shocks that appear to drive business cycles. But to the skewness is endogenous, it can be viewed instead as propagation and amplification mechanism. There has long been a search for mechanisms that can amplify and propagate small shocks that may initiate recessions (e.g., Summers *et al.* (1986)). In this sense, if skewness is an endogenous outcome that amplifies and extends business cycles, it reduces the size of the initial shock required to generate a recession.

Empirically, however, we do find evidence for at least some causal impact of skewness in driving business cycles. First, in manufacturing panel data for the US and internationally, we show that the skewness of shocks to firms' Total Factor Productivity (TFP) is

skewness (bottom panel of Figure A.3).

procyclical. To the extent that productivity is seen as a driving process, this suggests an exogenous component of skewness fluctuations. This also matches Dew-Becker (2022), who uses data from publicly traded firms to show that the firm-level option implied (i.e., forward-looking) skewness is procyclical. Second, in the COVID surveys, we see firms expected a more left-skewed growth rate of sales from April 2020 onwards, right at the onset of the pandemic, and a more right-skewed distribution during the onset of the recovery in early 2021. In this high-frequency monthly pandemic data, changes in skewness are contemporaneous with the start of the recession and recovery, consistent with a more causal story. Third, the changes in the skewness of firm growth are robust to subsamples of firm age, size, industry, or financial conditions, suggesting a broader aggregate change rather than some narrow channel of recessionary impact like adjustment costs, financial constraints, or industry heterogeneity. Finally, in local projection and vector auto regressions estimations, we show that shocks to skewness foreshadow declines in output even after controlling for a range of other first- and second-moment measures. Collectively, these results are suggestive that at least part of the decrease in micro-skewness is part of the recessionary process.

In the final part of the paper, we build a model to rationalize these results. We consider an economy with a large number of entrepreneurs with time-varying variance and skewness. Entrepreneurs are risk-averse, face a combination of convex and non-convex adjustment costs to capital, and can invest in their own firm and a risk-free asset. The productivity process is parameterized to match the skewness of the sales growth distribution in US firms over the business cycle. We find that a pure skewness shock—i.e., a one-standard-deviation decline in the skewness of firms' productivity shocks that maintains the mean and variance constant—reduces GDP by 0.4% after four quarters. The persistent drop in output is driven by a decline in capital investment from two forces. First, the presence of a fixed cost to capital adjustment creates a real options effect that reduces the incentives of firms to invest when skewness declines. This is a reflection of Bernanke (1983)'s classic *"Bad News Principle"*—that outcomes about the bad state of the world matter for option value to delay investment. Second, the drop in skewness makes capital riskier, inducing an increase in investment in the risk-free asset. Adding in a standard negative first and/or positive second moment shock leads to larger recessions.

**Contribution to the literature.** This paper is related to several strands of literature. First, our paper relates to the study of the effects of uncertainty on firms' decisions. Several papers have shown that an increase in uncertainty can have important macroeconomic implications in the presence of adjustment costs, risk aversion, or financial frictions, although there is still an active debate about the size and potential duration of the drop (see, for instance, Jurado *et al.* (2015), Ludvigson *et al.* (2021), and Bachmann *et al.* (2013)).[5] Our results are complementary to this literature as we show that the rise in the dispersion of firms' outcomes—a standard measure of uncertainty—results from a widening left tail and compressing right tail.

Second, several authors have suggested that rare disasters—presumably arising from an asymmetric distribution of shocks—can generate large fluctuations in economic activity, such as the Great Recession. Reviving the ideas introduced first by Rietz (1988), Barro (2006) considers a panel of countries to estimate the probability of large macroeconomic disasters and shows that these low-probability events can have substantial implications for aggregate economic activity and asset pricing. Several papers have confirmed the importance of fluctuations in disaster risk for aggregate economic activity.[6] The results of our paper can be seen as evidence that rare disasters also occur at the microeconomic level, and because firms are not typically perfectly insured against microeconomic risk, these firm-level disasters have large economic effects.

Third, our paper also contributes to a growing literature that studies the cyclical patterns of micro-skewness in individual labor earnings risk (e.g., Guvenen *et al.* (2014), Harmenberg and Sievertsen (2017), and Busch *et al.* (2022)), firm productivity (Kehrig, 2011), employment and sales growth (e.g., Davis and Haltiwanger (1992), Decker *et al.* (2015), among others), stock returns (e.g., Harvey and Siddique (2000), Oh and Wachter (2018), and Ferreira (2018), and several others), and option-implied skewness (e.g., Bollerslev and Todorov (2011), Conrad *et al.* (2013), an more recently Dew-Becker (2022)).[7] An older literature by Higson *et al.* (2002), Higson *et al.* (2004) and Döpke *et al.* (2005) document a counter-cyclical patterns of firm sales growth skewness using data from publicly traded firms in the US, the UK, and Germany respectively. This analysis, however, drops firms outside certain growth thresholds—for example, firms with sales changes outside ±25% in Higson *et al.* (2002)—which leads to highly cyclically sample selection, which reverses the underlying procyclical skewness in the full data set.

[5]Other important papers in the literature are Arellano *et al.* (2018), Fernandez-Villaverde *et al.* (2011), Schaal (2017), Bachmann and Bayer (2014), Gilchrist *et al.* (2014), Jurado *et al.* (2015), Leduc and Liu (2016), Basu and Bundick (2017), Berger *et al.* (2017), Kozeniauskas *et al.* (2018), Bloom *et al.* (2018), Bachmann *et al.* (2017). See also Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020) for recent surveys.

[6]See, for instance, Gabaix (2008, 2012), Gourio (2008, 2012, 2013), Wachter (2013), Kilic and Wachter (2015), Kozlowski *et al.* (2018, 2016), Venkateswaran *et al.* (2015), and Jordà *et al.* (2020).

[7]We also relate to the vast literature on the asymmetric behavior of the business cycle at the macro level (e.g., Colacito *et al.* (2016); McKay and Reis (2008); Veldkamp (2005); Segal *et al.* (2015) and several others), that tend to show sharp declines during recessions and slow recoveries during expansions.

In an important contribution, Ilut *et al.* (2018) focus on employment of large US manufacturing plants from 1972–2011 and show that employment growth skewness is procyclical but negative.[8] This paper, however, examines large manufacturing plants over a period of massive manufacturing contraction, spanning the Oil Crisis, the Rust-Belt recession, the China shock, and the Great Recession. Importantly, as Holmes (2011) reports, large manufacturing plants were the most likely to contract over this period as they were the most labor-intensive, with the average 1000+ employee plant shedding 60% of employment between 1977 and 2007 alone. Ilut *et al.* (2018) paper examines that period extended by 1972–1976 (the OPEC I recession) and 2008–2011 (the Great Recession), making large plants over this period a particularly extreme sample. Finally, Bachmann *et al.* (2017) use firm-level data on investment expectations to elicit the importance of firm shocks for investment decisions. They find that the skewness of investment innovations is acyclical. A challenge in interpreting this is since investment is curtailed at zero by adjustment costs this hinders inference on higher moments of the underlying driving process.

We believe our focus on multiple measures spanning sales, employment, productivity, and stock-returns, across multiple countries, industries, and time periods, combining administrative, accounting, and survey datasets, without sample trimming, paints a fuller picture of the cyclicality of firm growth skewness.

The rest of the paper is organized as follows. Section 2 describes the data we use and the statistics discussed in the empirical section. Section 3 shows the main empirical results of our paper, that is, that the skewness of several firm-level outcomes and productivity shocks is procyclical. Section 4 describes a simple model to account for the empirical evidence. Section 5 summarizes our results.

# 2 Data and Measurement

## 2.1 Data and Sample Selection

Our analysis is based on five large datasets that encompass firm- and establishment-level information for the United States and for a combined sample of 46 other countries.[9] The breadth of our dataset allows us to provide a detailed description of the cyclical patterns of the distribution of the growth rate of firm-level outcomes and firm productivity.

[8]The CMF surveys all 250,000 US plants every 5 years, but the ASM in the intervening years samples larger plants with 500+ employees and a random set of smaller plants. Hence, the panel-data containing plants in multiple panels is skewed towards much larger plants.

[9]Table B.3 in Appendix B.4 shows the list of countries in our dataset.

First, we extract panel data on firm employment from the US Census Bureau's LBD. The LBD provides high-quality measures of employment, wage bill, industry, and age for the entire US non-farm private sector, linked over time at the establishment level from 1976 to 2019. Skewness measures are extremely sensitive to outliers, so having several million observations per year for the aggregate sample, and for sub-sample cuts by age, size, industry and financial condition is critical. Starting in 1998, the LBD also contains information on firm-level revenues which we also use in our analysis (Haltiwanger *et al.*, 2019).[10] From the LBD, we calculate cross-sectional moments of the distribution of employment and sales growth within narrow firm population groups. The LBD contains over six million firms per year which, for measuring higher-order moments like skewness, is a major advantage. For consistency across datasets, and because our primary unit of analysis is the firm, we aggregate establishment data up to the US firm level.

Second, we obtain data on employment, revenues, capacity utilization, and productivity for a panel of manufacturing establishments by combining information from three Census data sets, the Census of Manufacturing (CM), the Annual Survey of Manufactures (ASM), and the Quarterly Survey of Plant Capacity Utilization (QPC), covering years from 1976 to 2019. From the merged dataset, we select establishments with at least ten years of non-missing, non-negative observations on employment and sales which, by the ASM methodology, oversamples larger establishments (and thus, implicitly, larger firms). We use this dataset to construct measures for firm-level (revenue) productivity and corresponding cross-sectional moments of the distribution of productivity shocks.

Third, we draw panel data of publicly traded firms from Compustat. Although this dataset contains mostly large established firms, it provides several additional variables that are helpful in our analysis. In particular, we use data on quarterly and annual sales, annual employment, and daily stock prices from 1970 to 2020, and we restrict attention to a sample of firms with more than ten years of data to reduce the types of compositional issues identified in Davis *et al.* (2006). Finally, by providing data on firms' entire global sales and employment—rather than just the US component in the Census—we can confirm that skewness of growth rates also pertains in firms' global sales, employment, and productivity data.[11]

[10] The revenue data at Census is collected at the firm-level and is available from 1998 to 2018. Approximately 30 percent of firms that have employment information but do not have revenue information. A significant fraction of these firms (about 25%) are in the NAIC 81 code (Other Services). The cross-sectional moments for the employment growth distribution do not change when we focus in the sample of firms that have employment and revenue data. See Haltiwanger *et al.* (2016) for additional details on this data.

[11] This is important because many large US firms have substantial activity outside the US, engaging

Fourth, we study whether the patterns we document for the United States are also observed in other both developed and developing. To that end, we use cross-country firm-level panel data on publicly traded firms containing sales and employment information between 1986 and 2018 from the Osiris dataset collected by Bureau van Dijk (BvD). In order to maintain homogeneous sampling criteria, we only consider firms with ten or more years of data. Additionally, we restrict our sample to country-year bins with more than one hundred firms. Our main results are based on an unbalanced panel of firms spanning 42 countries from 1984 to 2018. We complement this dataset with information on firm-level stock prices obtained from the Global Compustat dataset. Applying similar selection criteria, we obtain a sample of daily stock price information for firms in 29 countries from 1985 to 2020.

Finally, we obtain firm-level panel data from the Amadeus dataset also collected by the BvD. This dataset comprises a smaller sample of countries, for a shorter timespan, but with rich firm-level information for small and large firms, both publicly traded and privately held. In particular, Amadeus provides information on sales, employment, value added, capital, and labor input cost so that we can estimate firm-level TFP. Our sample contains information for 12 European countries starting in the mid 1990s.[12] Additional details on data construction, sample selection criteria, and moment calculation for each dataset used in our analysis can be found in Appendix B. A full replication packet for the empirical results of the paper can downloaded from the authors' website.

## 2.2 Measuring Dispersion and Skewness

For most of our results, we measure the growth rate of firm-level outcomes, such as employment and sales, by the arc-percent change between period $t$ and $t+k$, defined as $2 \times (x_{t+k} - x_t) / (x_{t+k} + x_t)$. This measure is bounded between –2, in the case a firm exits the market, and 2, in the case a firm enters the market. For labor productivity and stock returns, we focus on log changes between two years.

To further control for potential outliers, we measure dispersion and skewness using quantile-based measures. As we shall see, they also have magnitudes that are easy to interpret. Our measure of dispersion is the 90th to 10th percentiles differential, denoted by $P9010_t$, where $t$ is a quarter or a year depending on the dataset. Additionally, we use the 90th to 50th percentiles differential, $P9050_t$, and the 50th to 10th percentiles

in transfer pricing to shift activity (and profits) across borders, which could potentially be cyclical.

[12]Table A.1 in the Appendix summarizes the data sources and provides basic sample statistics for each of datasets we use in our analysis. Appendix Table B.3 shows a list of the countries we consider in our analysis and the data available for each of them.

differential, $P5010_t$, as measures of dispersion in the right and left tails, respectively. Finally, our preferred measure of skewness is the Kelley skewness (Kelley, 1947), which is defined as

$$KSK_t = \underbrace{\frac{P9050_t}{P9010_t}}_{\text{Right Tail Share}} - \underbrace{\frac{P5010_t}{P9010_t}}_{\text{Left Tail Share}} \in [-1, 1]. \tag{1}$$

This measure is useful as it provides a simple decomposition of the share of total dispersion that is accounted for by the left and the right tails of a distribution. A negative value of Kelley skewness indicates that the left tail accounts for more than one-half of the total dispersion and the distribution is negatively skewed, and vice versa for a positive value. Notice also that the Kelley skewness is invariant to 20 percent of the observations in the sample (the top and bottom 10 percent of the distribution are not considered). In principle, the Kelley skewness can be computed using any two symmetric percentiles, such as the 95th and 5th or 97.5th and 2.5th percentiles. Our results are robust to these choices and to using the third standardized moment.

# 3 Skewness over the Business Cycle

In this section, we show that firm employment and sales growth is positively skewed during expansions and negatively skewed during recessions in the United States (Section 3.1), across firms characteristics (Section 3.2), within industries (Section 3.3), and across several countries (Section 3.4).

## 3.1 US Macro Evidence

The first contribution of our paper is to show that the skewness of the growth rates of firm-level outcomes varies over time and is strongly procyclical. We start by considering the evolution of the Kelley skewness of the distribution of firm employment growth displayed in the panels of Figure 3 for two samples, the Census LBD covering all US firms and Compustat covering publicly traded US firms.[13] The top-left panel of Figure 3 shows, first, that the skewness of employment growth, on average, is positive and around 0.02 for most of the sample period. Second, the skewness of the distribution is strongly procyclical, declining from an average of 0.05 in expansionary periods to around –0.10 in recessions. This indicates that the share of the overall dispersion ($P9010_t$) accounted

[13] Results based on employment and revenue growth from LBD are weighted by lagged firm-level employment and revenues, respectively, to reflect aggregate importance. We weight using one-year lagged employment and sales to ensure the same values used to measure growth rates (i.e., one year forward minus current level) are not used in the weights. Unweighted results show similar empirical patterns.

for by firms *below* the median growth rate was about 47.5% during expansion periods, whereas this fraction rises to 55% percent during recessions. The right panel displays similar pattern across Compustat firms. More generally, we find that a decline in the GDP growth of one standard deviation is associated with a drop in the skewness of employment growth of 0.06 (Column 1 in Table I).[14]

Similarly, the bottom right panel of Figure 3 shows the cross-sectional skewness of annual sales growth for Compustat. We find that it is positive on average, and it declines to –0.07 during a typical recession, with an overall drop of 0.08 after a one-percent drop in GDP growth (Column 3 of Table I). We find the same pattern using revenue data from LBD albeit for a shorter period (bottom left panel) showing a clear decline in the skewness of firm-level revenue growth during the 2001 and 2008 recessions.[15]

One final macro result shown in Column (4) of Table I is that the skewness of firm-level stock returns is also highly correlated with GDP growth with a value of –0.12 during recessions and –0.05 during expansions. The change in the skewness of stock returns over the cycle suggests that the decrease in skewness of sales and employment growth is driven, at least in part, by a rise in negatively skewed external shocks (e.g., productivity or demand shocks) rather than skewed firm control variables (like investment or employment). In order to shed additional light on the cyclicality of the skewness of firms' shocks, in Section 3.3 we also directly test whether firm-level productivity shocks are left-skewed during recessions.

## 3.2 Heterogeneity

How does the cyclicality of skewness vary across firms characteristics? It is well-known, for instance, that small and young firms account for an oversized fraction of employment growth in the economy (Haltiwanger *et al.*, 2016) and are the most cyclical (Fort *et al.*, 2013). If that is the case, one would expect these firms to show a different cyclicality pattern in skewness relative to large, well-established firms. Similarly, financially constrained firms or firms with low past growth rates might be more exposed to large declines in employment or sales growth during recessions, hence accounting for the changes we observe in the data. Finally, some sectors might be more responsive than others to economic fluctuations, perhaps driving the aggregate changes in the skewness

[14] Table A.2 in the Appendix shows moments of the time series, cross-industry, and cross-country measures of skewness that we use in our analysis.

[15] Notice that there is a large decline in skewness of sales growth in 2014 without a corresponding recession which is mainly coming from large firms. In fact, Figure A.5 in the appendix shows using Census data that sales growth skewness declines mostly among the largest firms in the economy, most of which are publicly traded.

of firms' growth rates that we find in the data. To answer these questions, we exploit the large LBD sample size to divide firms into narrow population groups. Perhaps surprisingly, we find that distribution of firm growth shows similarly cyclical patterns, irrespective of sector, size, age, or other firm characteristics.

**Size, Age, and Wage Level.** The top panels of Figure 4 show that the skewness of employment growth is procyclical within firm-size groups.[16] Quantitatively, the skewness of employment growth declines almost the same during recessions, whether we look at small firms of less than 50 or 100 workers or if we look at large firms with 1,000 workers or more. We also find that skewness is equally procyclical among young firms of less than 5 years and for old established firms of more than 10 years of age, as shown in the bottom left panel. The same pattern is found across firms in sectors with different average wage level as shown in the bottom right panel of Figure 4.[17]

**Financial Constraints.** We use data from Compustat (balance sheet data is not available for the population of firms in LBD) to evaluate whether firms that are more financially constrained display larger procyclical fluctuations in skewness of firms growth relative to firms that are less constrained. We measure how financially constrained a firm is in two ways: (i) by lagged average debt-to-asset ratio (sum of short- and long-term debt divided market capitalization), and (ii) by the lagged average interest to EBIT ratio (interest expenses divided by earnings before interest and taxes).[18] Then within each year, we separate firms into two groups—above and below median—of the lagged measure of financial constraint and calculate the within-group skewness of sales and employment growth. The results are shown in the top panels of Figure 5. Although there is some heterogeneity across firms, the skewness of employment growth is procyclical for both types of firms, regardless of how constrained they are. We conclude that differences in access to credit across firms is not likely to be the main driver of the cyclical patterns observed in the skewness of firm-level growth.

[16] We classify firms according to their lagged employment. We use a lagged value of employment to avoid contemporaneous correlation between growth and firm-size classification since firms' growth is calculated between periods $t$ and $t+1$.

[17] Our results do not depend on the entry and exit of firms, and remain basically unchanged if we calculate the skewness of the log employment or sales growth—basically dropping firms with zero sales or employment—as shown in Figures A.4 and A.5 in the appendix, respectively.

[18] More precisely, we calculate the leverage of firm $j$ in period $t$ as long-term debt plus current liabilities divided by the total value of assets and the interest to EBIT ratio as total interest expenses divided by EBIT. For each of these measures, we calculate the lagged average value as $\overline{x}_{jt} = 0.5 \times (x_{j,t-1} + x_{j,t-2})$. We use a lagged value of leverage to reduce concerns about endogeneity between the measure of financial conditions and firms' growth. Appendix B discusses the construction of these measures in further detail.

**Recent Firm Growth.** Another possibility is that firms with poorer past performance are more likely to experience large negative drops in employment and sales once there is an aggregate shock (e.g., brick-and-mortar retailers doing poorly before the Covid pandemic might have experienced even larger declines in sales during the recession), hence explaining the negative change in skewness observed during recessions. To see if this is the case, we sort firms by their lagged average growth and calculate the skewness of employment and sales growth within each group. Although there are significant differences in the *levels* of skewness between high- and low-growth firms, the bottom panels of Figure 5 show that the skewness is similarly procyclical, dropping in recessions in a very similar fashion for both groups.[19]

These results indicate that the changes in the distribution of firm growth rates that are observed during a typical recession are not associated with a particular firm characteristic, such as size, age, leverage, or recent growth rates. As we show in the following section, these results are also robust across different industries and countries.

## 3.3 US Industry Evidence

We also examine industry-level business cycles and skewness. This analysis both provides more variation in our core US data and also confirms that our aggregate growth-skewness relationships are not driven directly by macro-factors like monetary or fiscal conditions. To do this, we take our US Census data and divide it into about 280, 4-digit NAICS sectors, and within each sector generate the annual Kelley skewness of firm employment growth rates and the average firm-level sales growth rates, so as to measure the aggregate conditions within the sector.

This is plotted in binscatter form in the top left panel of Figure 6. We see a strong upward slope: industries with higher-than-average mean sales growth rates also have significantly more positively skewed employment growth rates. The top right panel shows a similar result plotting the skewness of sales growth rates across firms against the industries' mean sales growth rates on the x-axis. Again, industries with strongly positive growth have heavily right-skewed firm growth rates while those with negative growth rates have heavily left-skewed firm growth rates. In terms of magnitudes, the top left panel of Figure 6 shows that when the average industry sales growth is –0.08, the Kelley skewness is around –0.15, indicating that 57% of the total dispersion in employment growth within an industry is accounted for by the left tail of the distribution. In contrast,

[19] We find the same results using Census LBD data for employment and revenue growth. These results are available inside the Census RDC servers.

when the average employment growth is 0.08, Kelley skewness is 0.20, so now it is the right tail that accounts for about 60% of the total dispersion.[20]

The evidence we have presented so far indicates that the skewness of the distribution of the growth rates of firm-level outcomes is procyclical. This could arise from shocks with time-varying higher-order moments (i.e., time-varying skewness in productivity or demand shocks) or the non-linear response of firms to aggregate shocks (like the Financial Crisis or COVID shock) or to (symmetric) idiosyncratic shocks which translate to non-linear responses (as in Ilut *et al.* (2018)). In this section, we provide evidence that the procyclical skewness is driven, in part, by firm-level idiosyncratic shocks that are left-skewed during recessions and right-skewed during expansions.

First, we use data for a sample of manufacturing establishments in the United States that combines records from the Census of Manufacturing and the ASM from 1976 to 2019. The Census Bureau also provides a readily available measure of establishment-level productivity, which we use in our analysis (Foster *et al.*, 2016; Cunningham *et al.*, 2021).[21] It is well known, however, that standard measures of productivity (such as the one provided by Census) might be affected by changes in capacity utilization (Basu, 1996). Intuitively, after a negative aggregate shock, firms might reduce the number of hours worked or the intensity of their equipment use, which might give the impression of a decline in productivity if not properly accounted for. If this response is heterogenous across firms, it might also lead to changes in the measured variance and skewness of firm-level growth.

To control for such changes, we regress productivity in the ASM on the number of hours worked in the firm and electricity use—common capacity measures—and use the residual as our firm-level productivity cleansed from capacity effects.[22] We then obtain a

[20]Similarly to the aggregate results discussed in Section 3.1, the change in the skewness of the within-industry distribution of firms' growth is driven by an asymmetric response of the right and left tails of the within-industry distribution of firm growth. This is clearly seen in Figure A.6 that shows that $P9050_t$ of employment growth is positively correlated with the within-industry mean sales growth, indicating periods in which the industry is growing above average, are periods in which the dispersion of the right tail expands and the opposite happens in periods in which the industry is growth below average. In contrast, the $P5010_t$ is negatively correlated with the industry cycle, increasing during periods of low economic activity within industry level. In term of sales growth—bottom panels of Figure A.6—the dispersion above the median is basically unresponsive to mean economic conditions within the sector whereas the left tails is negatively correlated. This contrasting response of the right and left tails of the within-industry distribution of firm growth is what drives the changes in the skewness.

[21]Since we do not have separated data on prices and quantities, our measure of productivity contain variation in physical productivity and demand, i.e., we measure revenue total factor productivity as in Foster *et al.* (2008).

[22]Our skewness results are not sensitive to this cleaning: the previous version of this paper (Salgado

measure of firms' productivity shocks, denoted by $\varepsilon_{i,t}$, from the residual of the following firm-level panel regression,

$$z_{i,t} = \beta_0 + \beta_1 z_{i,t-1} + \mu_i + \delta_t + \varepsilon_{i,t}, \tag{2}$$

where $z_{i,t}$ is the log-productivity of establishment $i$ in year $t$, $\mu_i$ is a firm fixed effect, and $\delta_t$ is a year fixed effect. We then calculate different moments of the distribution of $\hat{\varepsilon}_{i,t}$ within an industry-year bin. Because Census data only contains information about manufacturing establishments, here we divide our sample into 4-digit NAICS cells within a year, and we calculate the mean and the Kelley skewness of the productivity shock within each bin. As the bottom left panel of Figure 6 shows, the skewness of firms' shocks is negative in industries experiencing average sales declines.

Second, we examine data on stock returns, which like productivity, is another measure proxying the productivity and demand shocks hitting firms. The bottom right panel plots the stock returns at the NAICS 2-digit level for Compustat firms against their average sales growth rates. We see that in industries with positive sales growth rates that stock returns have right-skewed stock returns and in industries with negative growth rates stock returns have a left-skewed distribution.

These industry level results are also confirmed in columns (5) to (8) of Table I, which display a series of industry panel regressions in which the dependent variable is the Kelley skewness of the growth of employment, sales, productivity, and stock returns within an industry-year cell, and the explanatory variable is industry-level sales growth rates. In all cases, we find that periods of high sales growth are periods in which the skewness of firm growth is significantly positive. As shown in the bottom rows of Table I, the average skewness of firms' employment and sales growth is positive and becomes negative in industry recessions, defined here as years in which the within-industry growth is negative. The results are qualitatively similar for stock returns.[23]

## 3.4 Cross-Country Evidence

The second contribution of our paper is to investigate firm skewness in 46 other countries that are geographically and economically diverse, spreading over five continents

*et al.*, 2019) reports similar results which are not cleansed on capacity measures.

[23]We find a similar positive and statistically significant relationship between industry cycles and skewness when we consider each industry separately. Appendix Figure A.7 shows the slope coefficient of a set of within-industry time series regressions of the Kelley skewness of firms' growth on the within-industry average firm growth. Notice that, although there is substantial heterogeneity across industries, for all of them, the coefficient on the average firm growth is positive and economically and statistically significant.

including high-income countries (such as Germany, Japan, or France) and middle-income countries (such as Peru, Egypt, or Thailand).

Figure 7 binscatters four different country-by-year firm-level skewness measures against a within-country measure of the business cycle.[24] The top-left figure binscatters the Kelley skewness of the firm employment growth against country-by-year sales growth rates, revealing a strongly positive relationship. Going from a decline in average sales growth of –0.10 to an expansion of 0.1, the Kelley skewness moves from of –0.10 (implying that the *left* tail accounts for about 55% of the overall dispersion (P9010) in the employment growth distribution) to a Kelley skewness of 0.15 (indicating the it is the *right* tail now accounting for 57% of the total dispersion). The top right panel shows a similar result for measures of firm-level sale growth skewness, which is also strongly correlated with within-country economic conditions.

As with other firm-level growth measures, we find that the skewness of TFP shocks is procyclical in other countries. In this case, we use firm-level data for a sample of European countries obtained from the Amadeus dataset collected by the BvD for which we have rich enough information to measure firm-level (revenue) TFP shocks.[25] To facilitate the comparison with our previous results, the bottom left panel of Figure 7 shows a bin scatter plot in which each observation is a country-industry-year bin. In the $x$-axis, we plot the average sales growth within a cell, and in the $y$-axis, we plot the Kelley skewness. As in our previous results, we have controlled for country, industry, and year fixed effects. In this case, we also find that skewness of firm shocks and the within-industry average sales growth are positively correlated. In terms of magnitudes, an average decline of firms' sales 0.10 to –0.10 is associated with a decline of 0.02 in the skewness of the distribution.[26] The procyclicality of the skewness of firms' shocks

[24] To construct these figures, we have controlled for country- and time-fixed effects, so these results are not driven by fixed characteristics of the countries considered in the sample or by global shocks—such as the Great Recession—that can affect all countries at the same time.

[25] Our firm-level data from BvD Amadeus comprises information of small and large firms, both publicly traded and privately held from seventeen European countries, namely, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hungary, Ireland, Island, Italy, Netherlands, Norway, Poland, Portugal, Sweden, and Ukraine. For ten of these countries (Germany, Spain, Finland, France, Italy, Norway, Poland, Portugal, Sweden, and Ukraine) we have enough information to estimate firm-level TFP. Appendix B.5 describes in full detail the sample selection and estimation procedure.

[26] For our baseline results, we estimate firm-level TFP using a standard TFP estimation method as Olley and Pakes (1996). For further robustness, in Appendix B.5, we use five additional measures of productivity. In the first, we reestimate $z_{i,t}$, by running a firm-level OLS panel regression within each country; Second, we estimate $z_{i,t}$ through constant factor shares; Third, we estimate labor productivity by regressing firm log-value added on log-employment and a set of firm and time fixed effects. The fourth and fifth methods estimate $z_{i,t}$ using the procedures of Ackerberg *et al.* (2015) and Wooldridge (2009)

is robust to changes in the estimation method we use to calculate productivity, holds for each individual country in our sample, and it is robust to changes in the measure of within-industry cycle (see Appendix B).[27]

These results, together with the procyclical skewness of firms' stock returns reported in Section 3.1, show that the skewness of firms' growth is explained, at least in part, by shocks that are also left-skewed. This procyclical skewness could be driven, for instance, by rising bankruptcy during recessions, which would generate left-skewed demand shocks (e.g., if a major customer or supplier goes bankrupt, this will generate a large left-tail shock). The underlying driving process itself could also heterogeneously impact firms—that is, some firms lose badly in recessions and some firms gain heavily in booms.

In columns (9) to (12) of Table I, we repeat the cyclicality regression discussed above for the United States but this time exploiting the panel dimension of the cross-country dataset to assess the cyclicality of skewness in international data. The dependent variable is the skewness of employment growth, sales growth, productivity growth, or stock returns within a given country/year cell. The business cycle is captured by the log GDP per capita growth in the respective country, which we have rescaled to have a unit variance to facilitate the comparison with the rest of the results. The regressions also include a full set of time and country fixed effects to control for aggregate economic conditions that might affect all countries simultaneously or for fixed differences across countries. The results confirm our previous findings of procyclical skewness for all three variables with similar levels of statistical significance. Compared with the United States, the estimated coefficients is slightly lower across countries than for the US alone.

## 3.5 Macroeconomic Impact of Skewness Shocks

The results presented in the previous sections have shown that the skewness of firms' outcomes and firm-level productivity shocks is procyclical. We now move one step fur-

respectively, As we show in detail in Appendix B.5, these methods deliver similar results, qualitatively and quantitatively.

[27]It is also important to notice that our estimates of productivity do not directly control for potential non-linear responses of firms to aggregate economic conditions. This could be the case, for instance, if firms are differentially exposed to aggregate (mean) shocks. Such a relation might not be controlled for by simply adding a time and firm fixed effects as we do in Equation 2. To address this issue, in Appendix B.5.5, we consider two potential solutions. The first is to replace the year fixed effect by a flexible polynomial on aggregate GDP growth so as to purge the TFP residual from potential nonlinear responses to aggregate economic conditions. The second is to allow for firm-specific loadings into aggregate conditions (firm-specific $\beta's$ in Equation 2). Then, we re estimate 2 separately for each firm, and we include country-specific GDP growth instead of the year fixed effect. As we show in Appendix B.5.5, neither for these changes affect our main conclusions: within-industry skewness remains robustly procyclical.

ther and study whether fluctuations in the skewness of the distribution of firms' *shocks* are associated with fluctuations in aggregate economic activity. Identifying idiosyncratic shocks to firms is difficult, more so if one wants to study the aggregate effects of an unexpected change in the higher-order moments of the distribution of these shocks. Hence, in this section, we follow two complementary approaches, noting that while neither implies causality, they provide robust evidence that a decrease in firm skewness foreshadows declines in aggregate economic activity.

To provide such evidence, we first take a run a series of local projections (Jordà, 2005) to relate the effect of a skewness shock in quarter $t$ to the growth rate of macroeconomic variables in quarter $t+k$. We consider a standard set of macroeconomic variables such as GDP, Investment, Consumption, Employment, and the S&P500 index. We also include additional controls such as wages, CPI, and the Fed Funds Rate. Finally, to measure variations in the dispersion and skewness of firms' shocks, we use the P9010 and the Kelley skewness of the cross sectional distribution of daily returns in a given quarter.[28] We orthogonalize the $P9010_t$ and Kelley skewness from contemporaneous variations in the same variables used in the local projection and take the residuals as our measures of shocks to dispersion and skewness.

The top left panel of Figure 8 shows that a one-standard-deviation drop in the skewness of firms' shock is associated with a significant and persistent decline in quarterly GDP. In particular, four quarters after the skewness shock, GDP declines by about 0.5% and only recovers to its pre-shock level after 12 quarters. Considering that during the Great Recession, the skewness of the quarter-to-quarter daily returns declined by two standard deviations, we conclude that changes in the skewness of return can have a significant impact on macroeconomic activity. We also find a significant response of aggregate employment that drops by about 0.3% four quarters after the shock, and investment, which drops by 2% after the shock. Finally, aggregate consumption shows a similar pattern, declining by about 0.4% after four quarters.

Our results are robust to a series of alternative specifications. One important one is separating positive and negative shocks to skewness (Figure 9). These results reveal that negative skewness shocks predict recessions and positive shocks predict expansions, highlighting the symmetric nature of this relationship. We also examine different measures of Kelley skewness (e.g., using alternative percentiles), use the third standardized

[28]We calculate the measures of dispersion and skewness weighting daily returns by the employment of each firm, although our results do not change significantly if we do not use any weight. See additional details on Appendix C.

moment, and evaluate the impact of a skewness shock using a standard VAR (Appendix Figure C.13), finding similar results.

Finally, we exploit cross-country/industry variation in the skewness of firm-level TFP *shocks* as estimated in Section 3.3 using data from the BvD Amadeus dataset to evaluate its impact on firm-level growth. In particular, we run a set of firm-level OLS panel regressions of the form

$$x_{it}^{jk} = \beta_0 + \beta_1 KSK_t^{jk} + X_{it}^{jk}\Gamma_t + \epsilon_{it}^{jk},$$

in which the dependent variable, $x_{it}^{jk}$, is a measure of firm growth, such as sales growth, employment growth, or investment; In this case, $KSK_t^{jk}$ is the cross-sectional skewness of firms' TFP *shocks* within an industry $j$ in country $k$ in year $t$. The set of controls in $X_{it}^{jk}$ include time and firm fixed effects (so as to account for aggregate fluctuations and observed and unobserved differences across firms) and several firm-level controls (e.g., size, age, past firm growth, etc.). We also include in $X_{it}^{jk}$ the cross-sectional median and standard deviation of firms' TFP shocks so that our results do not confound variations in the skewness of shocks with variations in the first and second moments of the distribution of shocks.

The results are shown in Table II. The first three columns shows that a change in the within-country/industry skewness of firms' TFP shocks is significantly associated with firms' sales. Quantitatively, the results indicate that a decline in the within-country/industry Kelley skewness of firms shocks of 0.1 foreshadows an average decline in firms' annual sales of 2.7%. The movement of employment growth and investment—measured by the log-change in firm fixed capital stock—are smaller in magnitude but still economically significant: column (6) shows that employment drops by 1.3% whereas column (9) indicates that investment drops by 0.8% after a decline in the Kelley skewness of firms' shocks of 0.1.

# 4 A Quantitative Exploration

In this section, we briefly describe a quantitative model that illustrates the potential macroeconomic impact of shocks to the skewness of firms' productivity. We make two modeling choices that are important in generating our results. First, we consider a model with risk-averse entrepreneurs so as to capture the fact that the vast majority of firms in our sample are small- and medium-size firms. Entrepreneurs in our model are exposed to aggregate and idiosyncratic risk that cannot be fully diversified. This seems plausible as very few businesses are able to insure fully (or even partially) against the risks they face, and since the managers of most firms have significant equity stakes in their businesses, they are exposed to business risk.[29] Second, as is standard in the literature (e.g., Quadrini (2000); Cagetti and De Nardi (2006)), we assume that markets are incomplete and entrepreneurs can save in a risk-free asset and on capital, which they use in production. The risk-free return can be thought of as a government bond or a foreign asset, which provides a return that is independent of entrepreneurs' idiosyncratic risk. In what follows, we provide a succinct description of the key elements of the model and relegate the rest of the exposition and details of the quantitative application to Appendix D.

**Entrepreneurs and Households.** We consider a large number of risk-averse entrepreneurs that supply one unit of labor to their own firm, invest in capital (used in their own firm and is subject to adjustment costs) and in a risk-free asset that pays a fixed interest rate. Entrepreneurs value consumption by means of a standard CRRA utility function given by $u(c) = c^{1-\xi}/(1-\xi)$. They operate a standard Cobb-Douglas production function with decreasing returns to scale that uses capital and labor to produce a homogeneous good. Finally, entrepreneurs' production is subject an aggregate shock—which follows a standard AR(1) process with Gaussian innovations—and an idiosyncratic shock—which follows a AR(1) process with innovations subject to time-varying variance and time-varying skewness.

Labor is supplied by a representative hand-to-mouth household that chooses how much to consume and to work so as to maximize a standard separable utility given by $U(C_t, N_t) = \left\{\frac{C_t^{1-\xi}}{1-\xi} - \psi\frac{N_t^{1-\psi}}{1-\psi}\right\}$ subject to $C_t \leq w_t N_t$.

**Parameters and Estimation.** Most of the model parameters, which we summarize in Table D.1, are standard in the macro literature, and we take them from existing estimates. The parameters governing the stochastic process of firms' productivity, however, are novel to our analysis, and we use the method of simulated moments to estimate them. To capture time-varying risk, we assume that the economy transitions between two aggregate states, a "low-risk" state, which corresponds to periods in which the variance

[29] Even in very large publicly traded firms in the United States, top executives own substantial equity stakes on the firms they manage (Eisfeldt *et al.*, 2021; Smith *et al.*, 2019) and a large fraction of top wealth owners—who also manage large firms—receive income from businesses (Boar *et al.*, 2022). In addition, Panousi and Papanikolaou (2012) show that managers and CEO respond to increases in firm-level idiosyncratic risk by reducing firm investment, and such response increases with the share of ownership that the manager or CEO has in the firm, which is consistent with models where managers act as risk averse agents. We further discuss these issues in Appendix D.

of the innovations of the idiosyncratic shocks is low and the skewness is positive (i.e., expansion periods), an a "high-risk" state, which corresponds to periods in which the variance of the innovations of the idiosyncratic shocks is high and the skewness is negative (i.e., recessions). Low- and high-risk states alternate following a first-order Markov process and we estimate the skewness of the underlying productivity process so that the model matches the observed variation in the revenue growth distribution observed in the LBD (e.g., a decline in the skewness of sales growth from 0.10 to –0.11 in Figure 1). Importantly, and consistent with the evidence from firm-level expectations presented in Figure 2, we have assumed that the distribution of innovations in period $t$ depends on the values of the variance and skewness observed in period $t-1$. This timing captures the "news shock" aspect of firm-level risks in the model: an increase in dispersion or a decline in the skewness of firms' shocks represents news about the characteristics of the distribution of innovations in the future but not a concurrent change in the distribution from which the realizations of idiosyncratic shocks are drawn.

**The Macroeconomic Effect of a Skewness Shock.** To quantify the impact of a decrease in the skewness of firms' productivity shocks, we consider an increase in risk that reduces the skewness (from positive to negative) while keeping the mean and variance of the idiosyncratic productivity process constant at their low-risk level. In that way, we the separate the effect of a skewness shock from a symmetric increase in dispersion, as is typically done in the uncertainty literature.[30]

Figure 10a shows the impact of a one-standard-deviation skewness shock. We find that GDP declines by about 0.4% four quarters after a skewness shock and 0.45% after eight quarters.[31] This is a significant decline in aggregate economic activity considering that only the shape of the distribution of firm-level shocks has changed. Similar to our empirical estimates, the decline in output is quite persistent, staying below its pre-shock level even after twelve quarters after the shock. Capital investment—the right y-axis in Figure 10a—declines by 5% during the first quarter after the shock and stays below its pre-shock level for several quarters. Finally, consumption declines rapidly in response to the decrease in the skewness of firm-level shocks, dropping by around 0.3% relative to

[30] To make this comparison, we reestimate the parameters of the firms' idiosyncratic productivity process to separate the changes in dispersion (a symmetric increase in risk) from changes in dispersion and skewness (an asymmetric increase in risk). Table D.3 in the Appendix shows the estimation targets for each case. Appendix (D.7) shows in detail how our simulations separate changes in the skewness from changes in the mean and variance of shocks.

[31] We can also translate this figure into the effect going from a typical expansion to recession in our sample (a decline in skewness of sales growth from 0.10 to –0.11), which implies an approximately 1.7% decline in GDP due to the skewness shock.

its pre-shock level. Behind the decline in investment and consumption after a skewness shock is the significant increase in savings in the risk free assets. As firms become riskier, entrepreneurs decide to increase their savings; however, the increase in risk is due to a decrease in firms' skewness. As a consequence, entrepreneurs decide to move their saving into the risk free asset, driving a joint decline in consumption and investment.

As we show in the previous sections, recessions are characterized by an increase in dispersion and a decrease in the skewness of firms shocks. This case is shown in Figure 10b that follows the macroeconomic impact of a concurrent increase in the variance and a decrease in the skewness of firms' productivity. In this case, GDP declines by slightly more than 0.4% four quarters after the shock. Relative to the pure skewness shock, this (small) additional decline in output is explained by a larger decline in investment and consumption and a surge in investment in the risk-free asset. The combined effect of variance and skewness accelerates the recovery after the shock as output starts to recover rapidly six quarters after the shock. Hence, our results suggest that the joint effect of an increase in dispersion and a decline in skewness of firms' productivity can generate aggregate dynamics that are similar to those observed in a typical recession.

Two main channels drive the results in our model. The first is the real-option effect that increases the value of waiting when uncertainty increases. Notice that our empirical evidence does not indicate an overshoot of macroeconomic activity, which is typically observed after an uncertainty shock. Instead, we find a persistent decline of economic activity. Our model rationalized this in two ways. First, part of the overshoot observed after an uncertainty shock—which implies a symmetric increase in uncertainty—derives from firms that experience a larger than expected productivity shock, drawn from the now more dispersed right tail of the productivity distribution. If the increase in uncertainty, however, is accompanied by a decrease in the skewness—as we observe in the typical recession—the left tail of the productivity distribution expands while the right tail compresses and hence the number of firms experiencing a large positive shock declines. In other words, a skewness shock reduces the impact of the *Oi-Hartman-Abel* effect, which is observed after a *symmetric* increase in dispersion of firm productivity. Second, after a skewness shock, risk averse entrepreneurs move their wealth from their riskier firms to the risk-free asset, reducing capital investment even further, and since uncertainty remains elevated after a skewness shock for some periods, they increase the capital invested on their firms slowly. These two effects tend to ameliorate the overshoot of macroeconomic activity observed after an uncertainty shock.

# 5 Conclusions

This paper studies how the distribution of the growth rate of firm-level variables changes over the business cycle. Using firm-level panel data for the United States from Census and non-Census datasets and firm-level panel data for almost forty other countries, we reach three main conclusions. First, recessions are characterized by a large drop in the skewness of firm-level employment, sales and productivity growth. Expansions, in contrast, are characterized by positive skewness in firm employment, sales and productivity growth. Second, the decline in the skewness of firms' outcomes is a phenomenon observed not only in the United States but also in other countries, both developed and developing, and within industries. Third, empirical and model-based estimates indicate that that shocks to the skewness of firm-level innovations predict future declines in macroeconomic activity. Taken together, these results suggest that the increase in firm-level disaster risk observed during the typical recession can play a major role in accounting for business cycles fluctuations.

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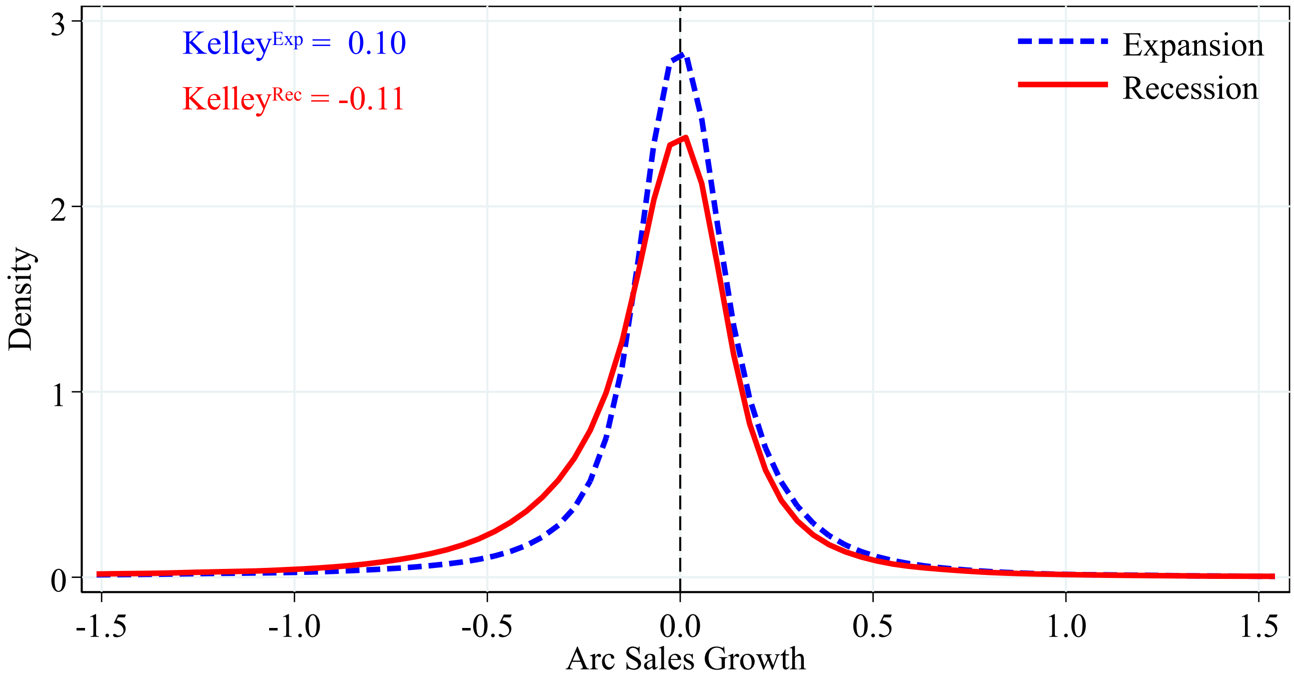
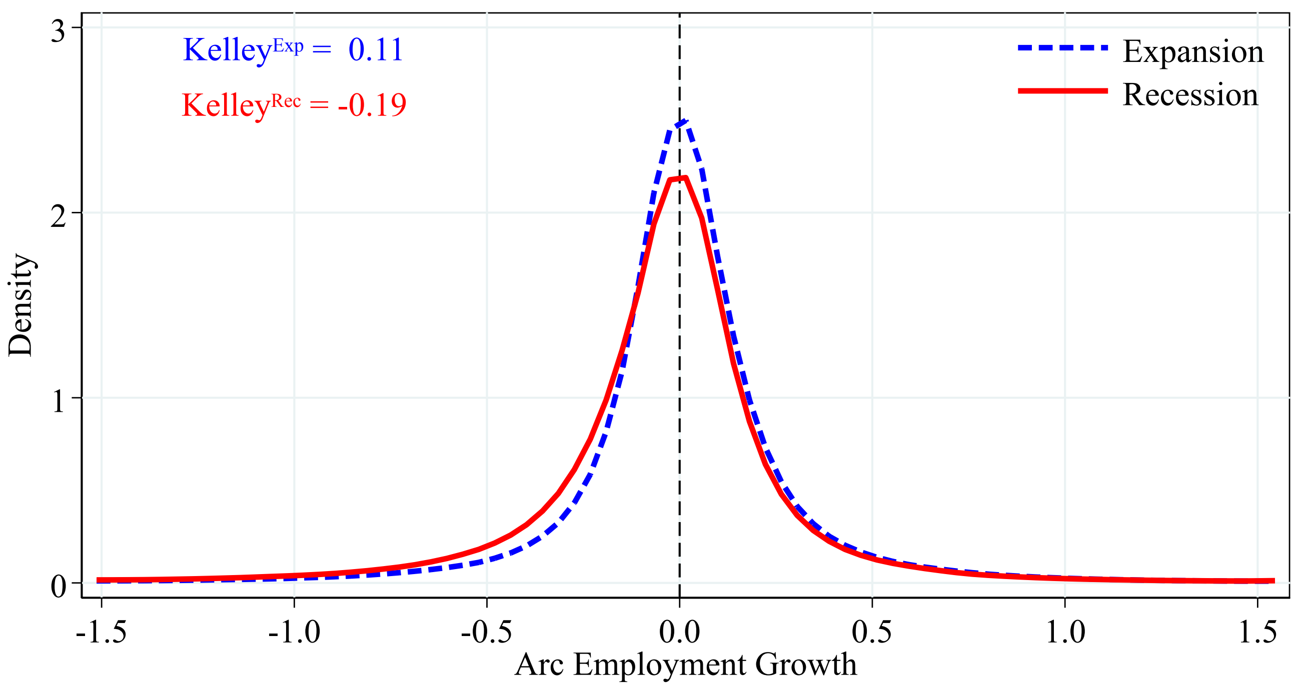
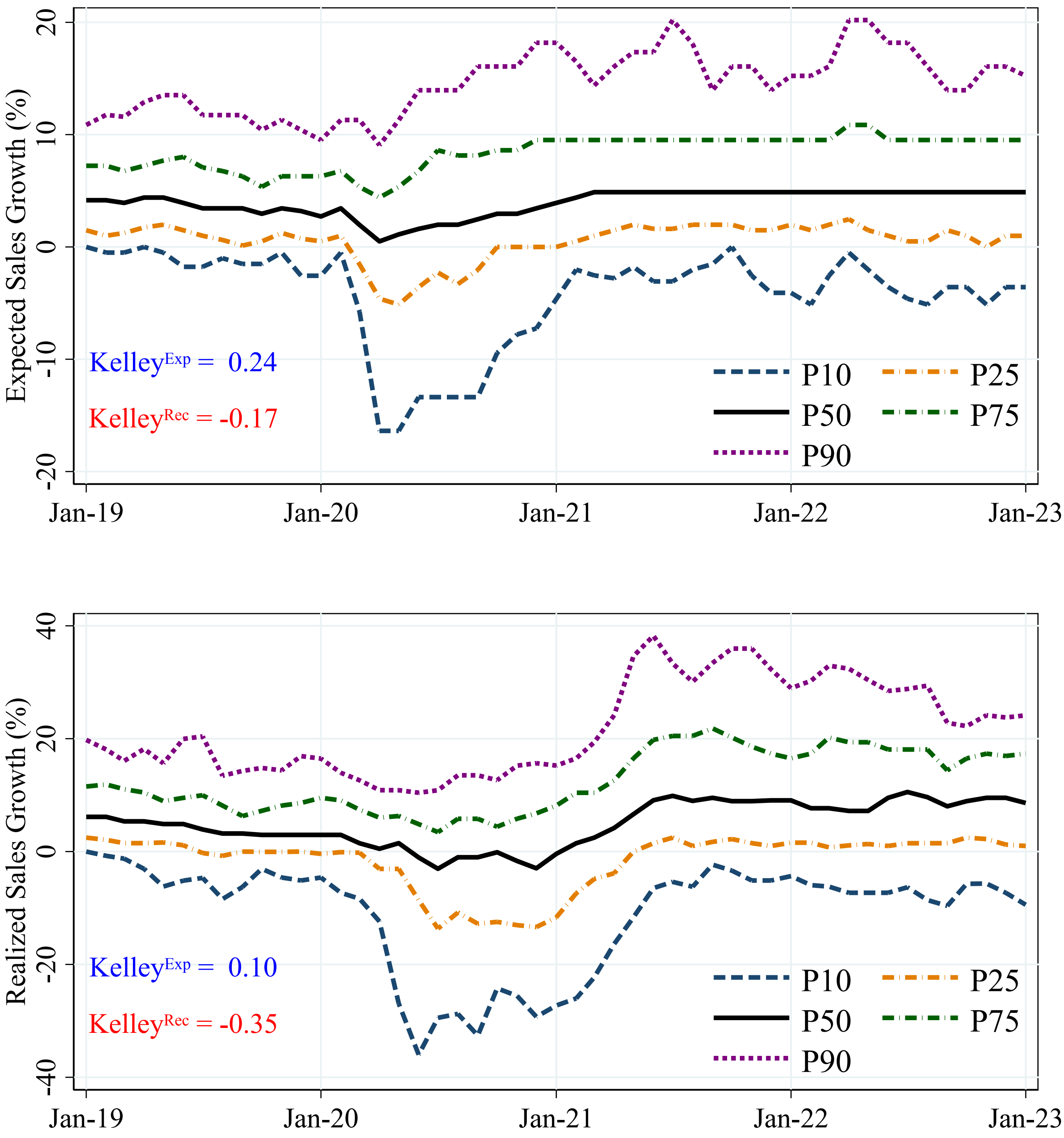
Figure 1 – Skewness of firm sales and employment growth is pro-cyclical



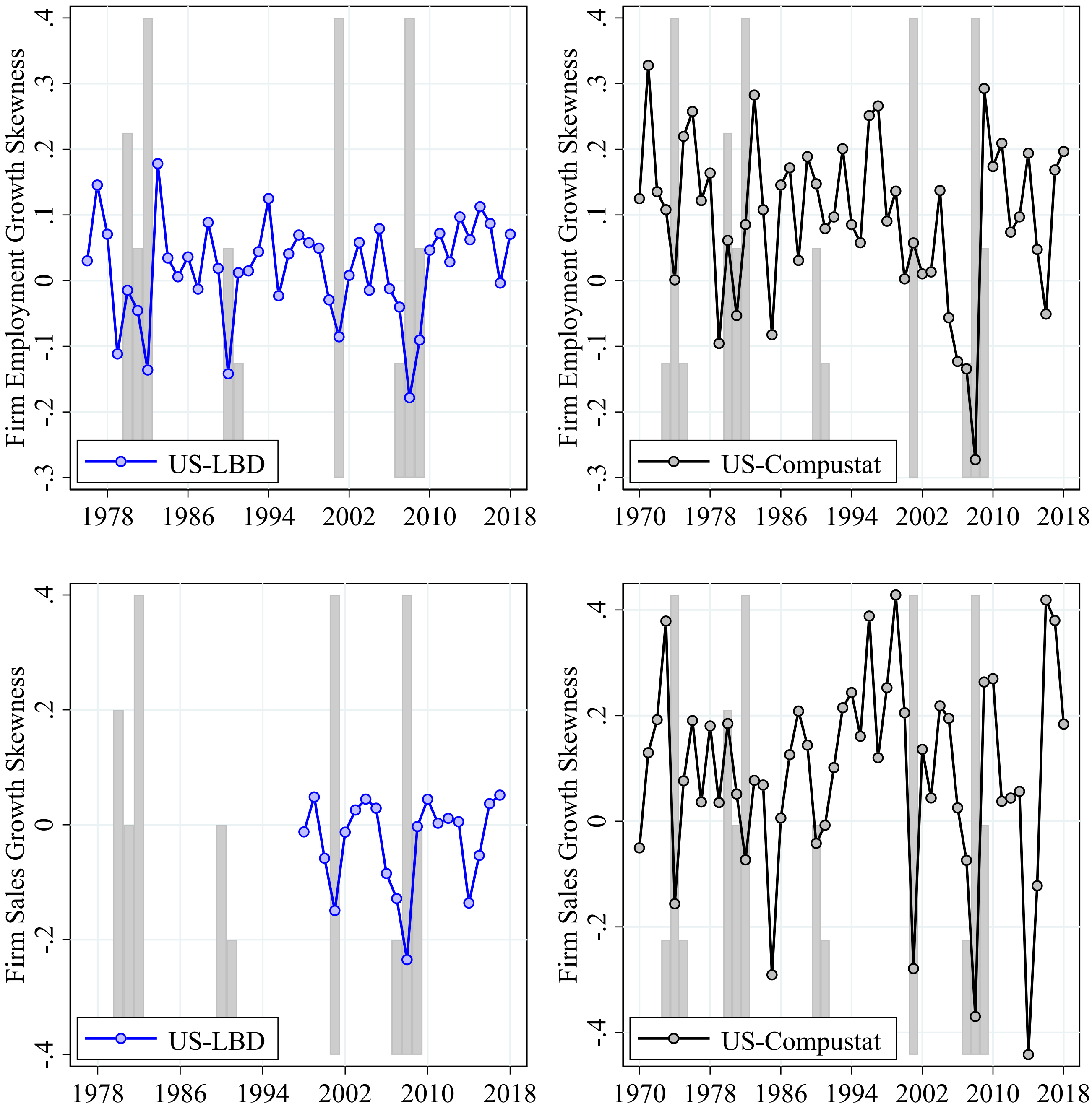

Note: The top panel (bottom panel) shows the sales-weighted (employment-weighted) empirical density of the distribution of firms' arc-sales growth (arc-employment growth) between years $t$ and $t+1$ constructed from the US Census' Longitudinal Business Database (LBD) for firms (where firm employment sums across all establishments within the firm) with 20 employees or more. Arc-sales in period $t$ is calculated as $(x_{t+1} - x_t) / (0.5 \times (x_{t+1} - x_t))$. Each density has been adjusted to have a median of zero. The blue-dashed line shows the density of a pooled sample of expansion years (2003 to 2006 and 2010 to 2014) for a total of approximately 590,000 firm-year observations; the red-solid line shows the density of a pooled sample of recession years (2001 and 2008) for a total of approximately 570,000 firm-year observations. Kelley refers to the Kelley Skewness, calculated as $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] / (P90 - P10)$ where $P90, P50, P10$ are the weighted percentiles of corresponding distribution.

Figure 2 – COVID Skewness of Expected and Realized Sales Growth



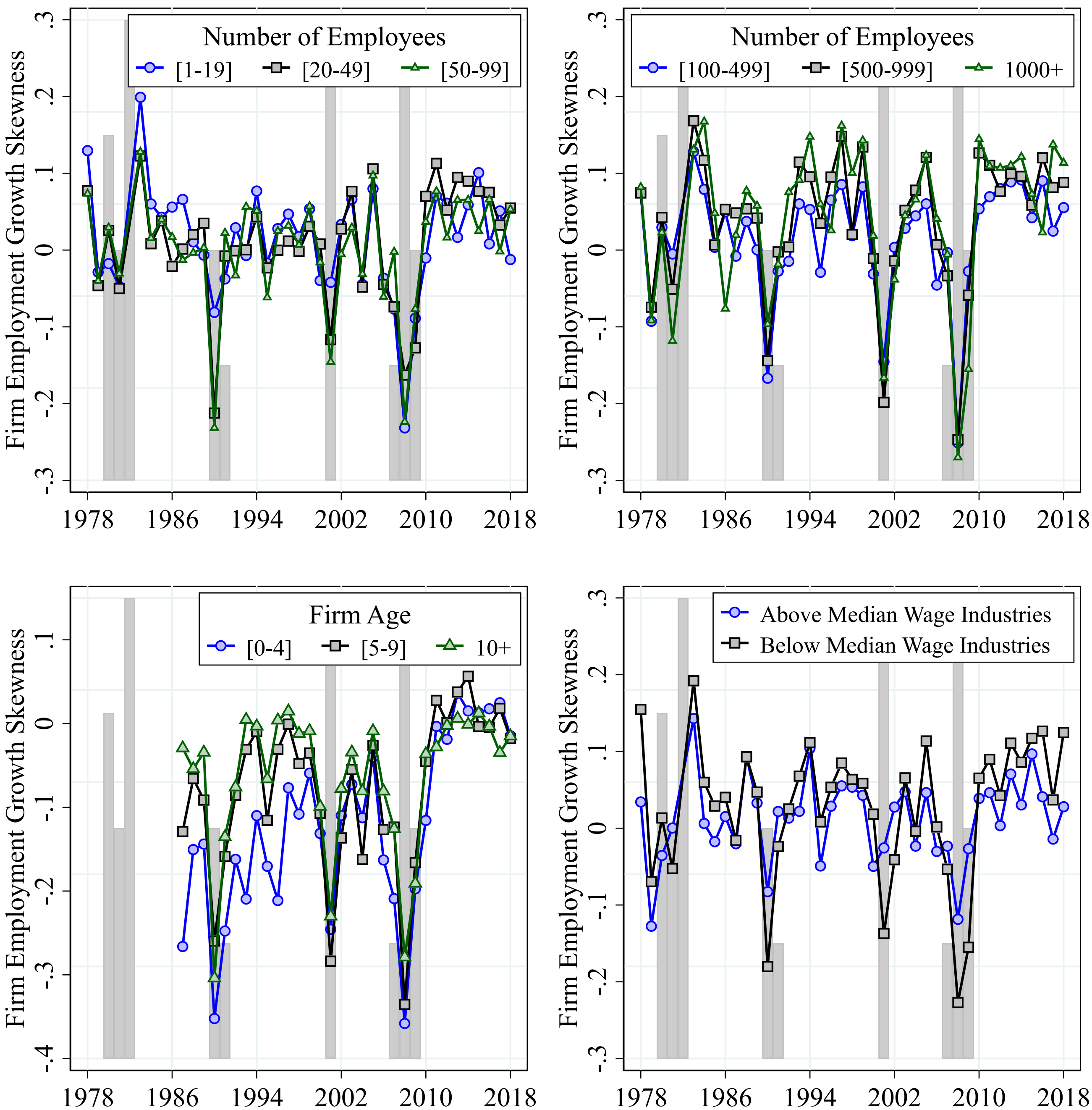

Note: Figure 2 shows the distribution of expected and realized annual sales growth for a representative sample of public and private firms in the United States. Data is collected by the Federal Reserve Bank of Atlanta with about 500 observations per month covering all major sectors and firm size groups. Kelley Skewness is calculated as $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] \,/\, (P90 - P10)$. Because of the two-panel rotating month survey structure, the data is shown as a two-month lag moving average. Additional information on the methods and data collection can be found at www.atlantafed.org/research/surveys/business-uncertainty.

Figure 3 – Skewness of Firm Growth is Procyclical from 1970 to 2018



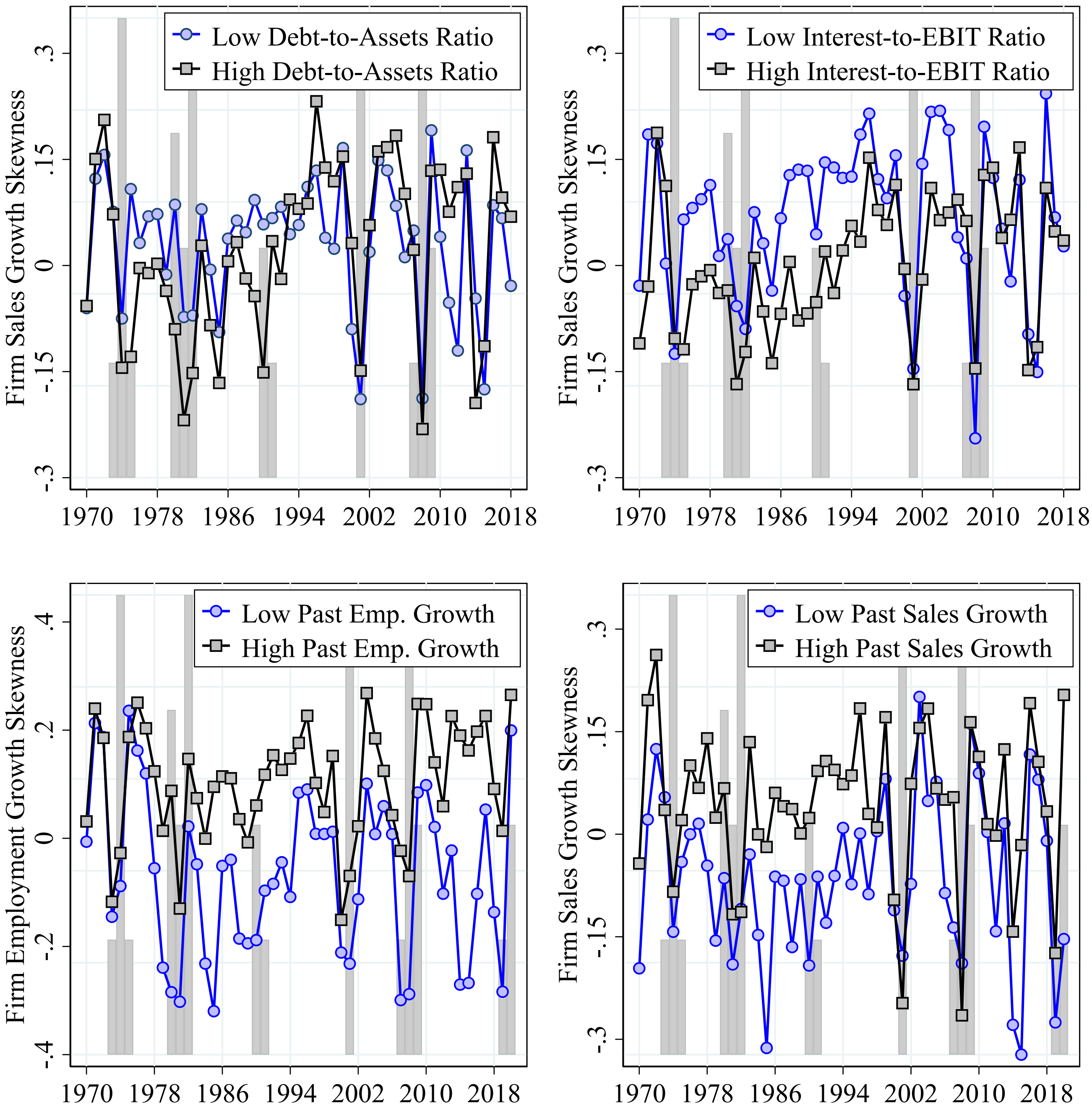

Note: Figure 3 shows the time series of the cross-sectional Kelley skewness of the distribution of firm employment growth (top panels) and firm sales growth (bottom panels). Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] / (P90 - P10)$ where $P90, P50, P10$ are (lag) employment-weighted (top panels) and (lag) sales-weighted percentiles of the corresponding distributions. The left-panel uses data from the US Census' Longitudinal Business Database (LBD) containing an average of about 4.6 million firm observations per year. Data from sales in LBD is only available after 1998. The right-panel data from Compustat contains an average of about 4,500 firms observations per year for a sample of firms with 10+ years of data. Shaded areas are the share of NBER recession quarters within a year.

Figure 4 – Skewness of Employment Growth is Procyclical by Firm Groups



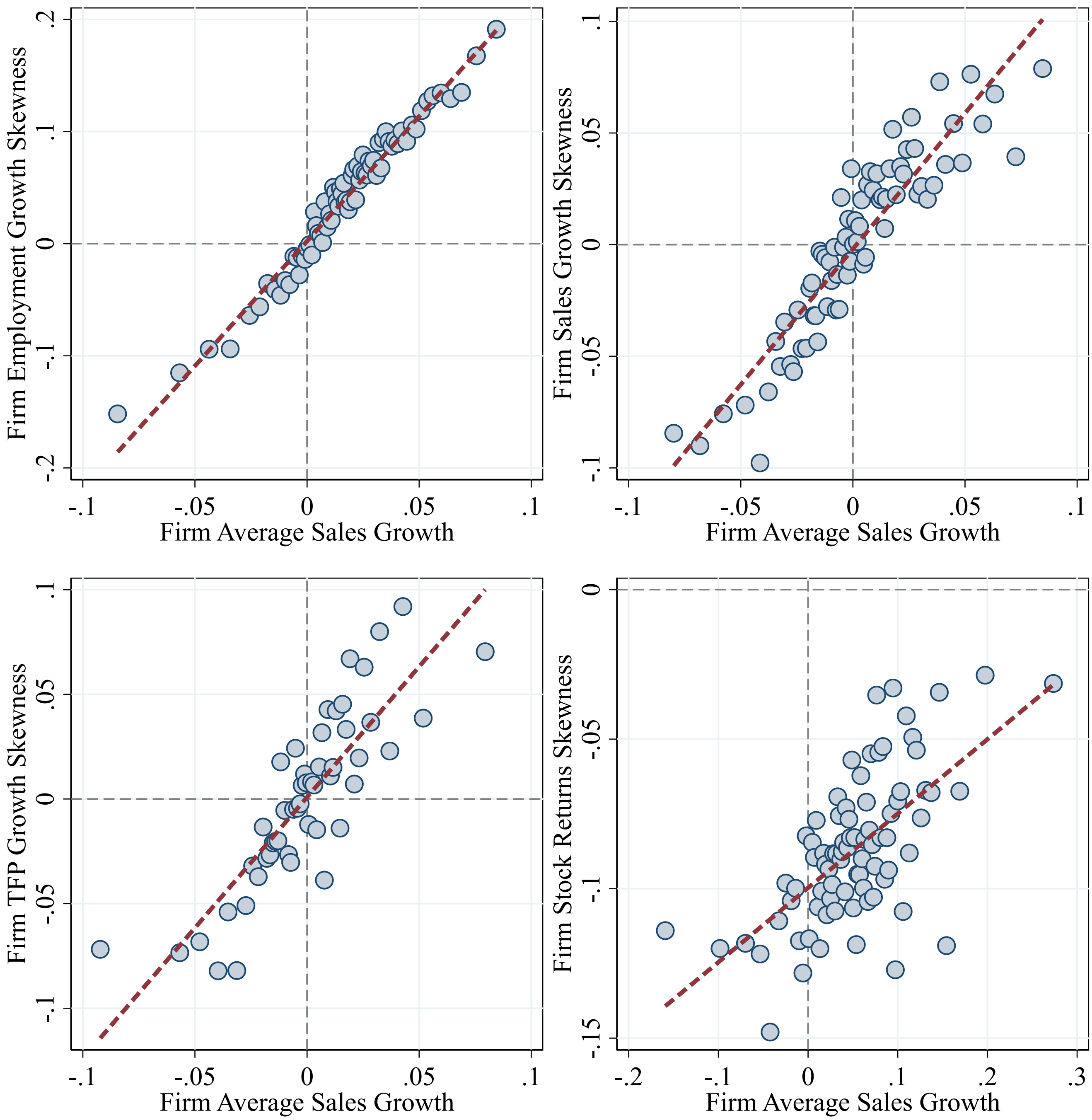

Note: Figure 4 shows the time series of the cross-sectional Kelley skewness of the distribution of firm employment growth for different firm groups using data from the US Census' Longitudinal Business Database (LBD), containing an average of about 4.6 million firm observations per year. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] / (P90 - P10)$ where $P90, P50, P10$ are (lag) employment-weighted percentiles of the employment growth distribution. Age groups start in 1987 since it is the first year in which we can identify firms with 10 or more years of age. Age for firms present in LBD in 1976 is not available. Number of employees is the total number of workers of a firm across all establishments. Shaded areas are the share of NBER recession quarters within a year.

Figure 5 – Firm Growth Skewness and Financial Conditions



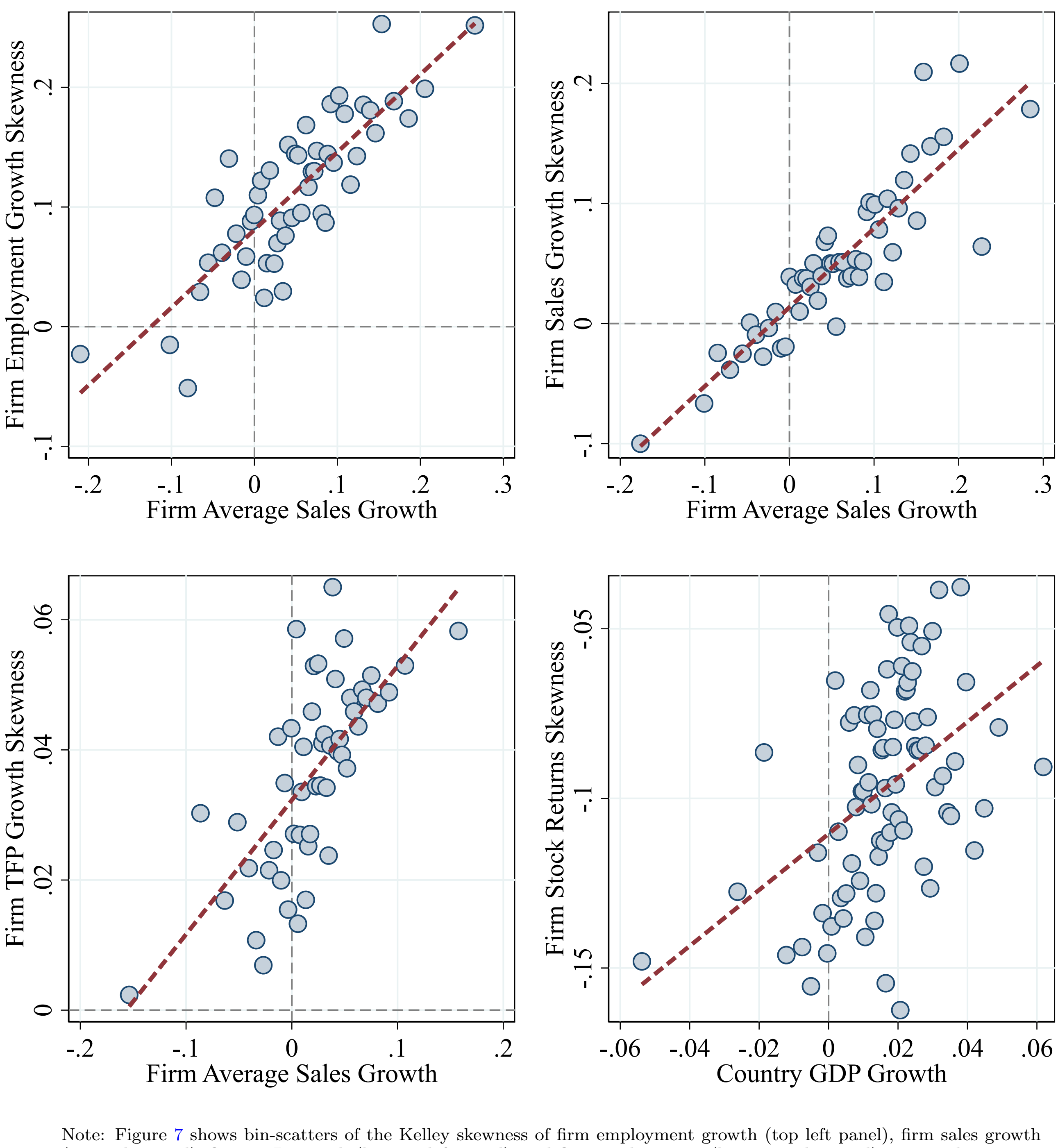

Note: Figure 5 shows the time series of the cross-sectional Kelley skewness of the distribution of firm employment growth (top and bottom left panels) and sales growth (bottom right panel) for different firm groups using data from Compustat for firms with 10+ years for a sample of about 4,500 firms observations per year. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] / (P90 - P10)$ where $P90, P50, P10$ are (lag) employment-weighted percentiles of the employment growth distribution. Debt in the top left panel is the sum of long and short term debt. In the top right panel, interest is the total interest paid in long- and short-term debt. In all panels, we separate firms in Low and High relative to the median of the (lagged) distribution (e.g., Low Past Sales Growth are firms below the median of the lag sales growth distribution). Shaded areas are the share of NBER recession quarters within a year. The elasticity of the skewness of Low (High) Debt-to-Assets Ratio firms is 0.042 (0.062 respectively). On the top right panel, for Low (High) Interest-to-EBIT Ratio firms, the elasticity is 0.057 (0.043 respectively). In the bottom left, for Low (High) past growth, it is 0.10 (0.061). On the right panel, Low (High) Past Growth firms, it is 0.051 (0.041). All coefficient are significant at the 1% level.

Figure 6 – Skewness versus Average Growth Across US Industries



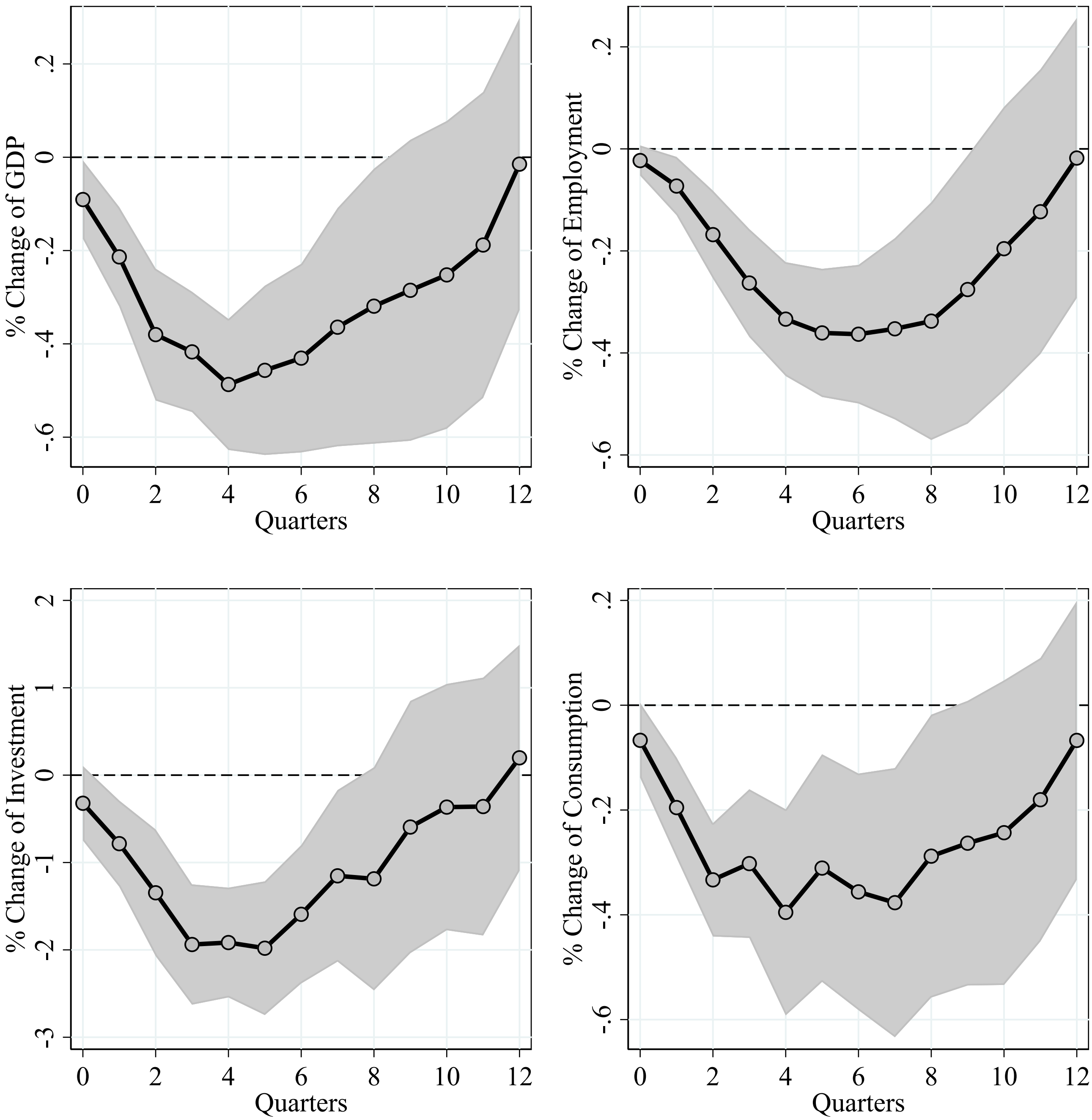

Note: Figure 6 shows bin-scatters of the Kelley skewness of firm employment growth (top left panel), firm sales growth (top right panel), firm TFP growth (bottom left panel), and firm stock returns (bottom right panel) against the average sales growth within an industry-year. Top panels use data from the US Census' Longitudinal Business Database (LBD). Industries are defined as 4-digit NAICS for 284 industries covering the years 1998 to 2018 (sales data only available from 1998 onwards in LBD), for an average sample of about 4.6 million firms per year. The bottom left panel uses data from the US Census' Annual Survey of Manufacturing (ASM) and the Census' Quarterly Survey of Plant Capacity Utilization (QPC). Industries defined at the 4-digit NAICS for 78 industries covering the years 1976 to 2018, for an average sample of about 30,000 establishments per year. We use data from QPC to adjust our measures of TFP from changes in capacity utilization. The bottom right panel uses data from CRSP for stock returns and Compustat for sales growth. Industries are defined as 2-digits NAICS for 81 industries covering the years 1974 to 2019, for an average sample of about 4,500 firms per year. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = \left[(P90 - P50) - (P50 - P10)\right] / (P90 - P10)$ where $P90, P50, P10$ are percentiles of the corresponding distributions. Bin-scatters control for industry and year fixed effects.

Figure 7 – Skewness versus Average Growth Across Countries



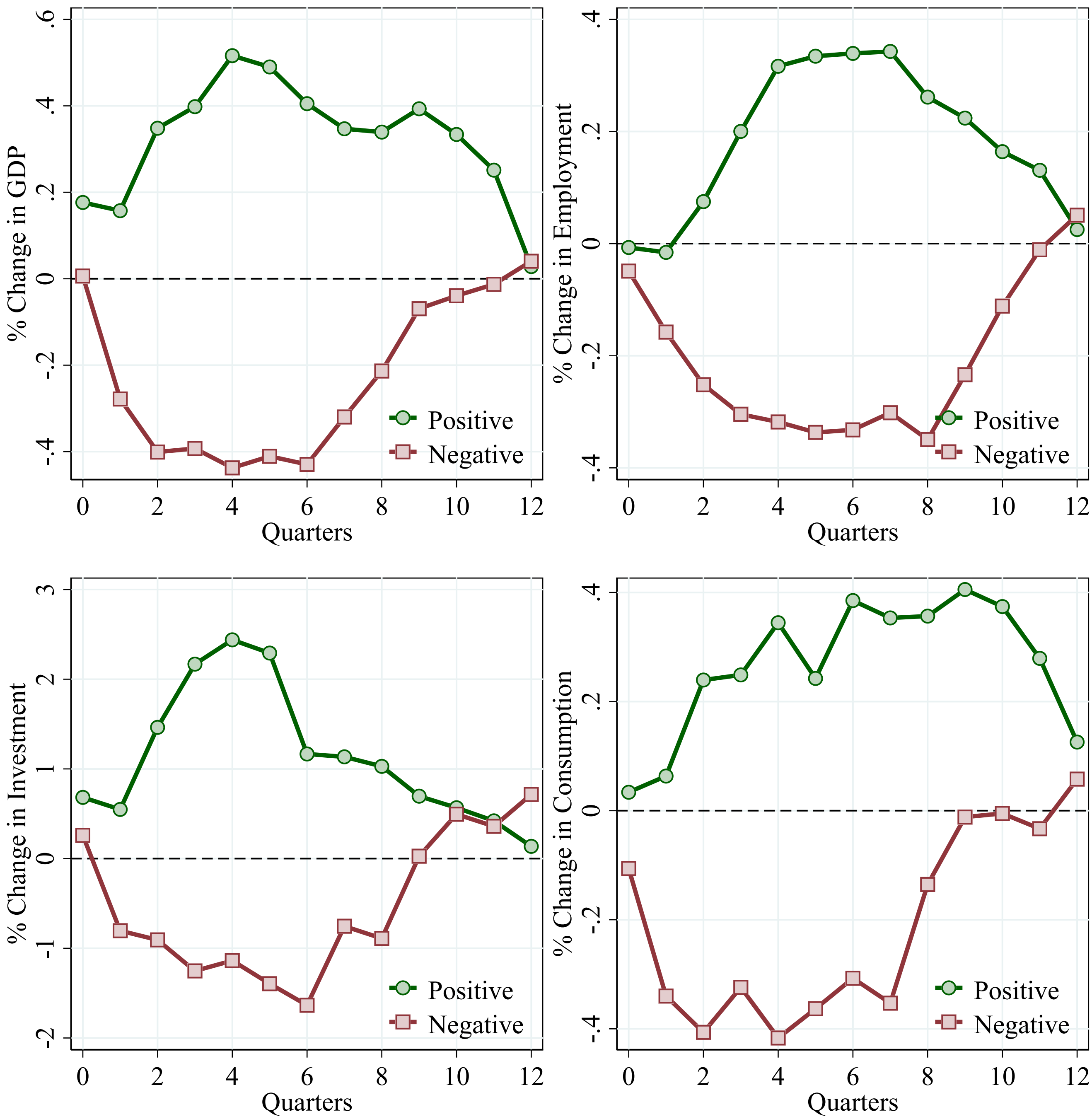

Note: Figure 7 shows bin-scatters of the Kelley skewness of firm employment growth (top left panel), firm sales growth (top right panel), firm TFP growth (bottom left panel), and firm stock returns (bottom right panel) against the average sales growth within a country-year (top panels), within a country-industry-year (bottom left panel), and quarterly GDP growth (bottom right). Data on quarterly sales growth for the same sample is not available. Top panels use data from from Osiris for 42 countries covering years 1984 to 2018, for an average sample of about 11,700 firms per year. The bottom left panel uses data from Amadeus with 2-digit industries in 10 countries covering years 1999 to 2018, for a sample of about 320 thousand firms per year. The bottom right panel uses data from from Global Compustat (bottom right) for stock returns and OECD for GDP growth for 31 countries covering the years 1985 to 2020 for a sample of about 38 thousands firms per years. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] \,/\, (P90 - P10)$ where $P90, P50, P10$ are the percentiles of the corresponding distributions. Bin-scatters control for country (and industry) and year fixed effects.

Figure 8 – Skewness Shocks Predict drop in Macro Activity




Note: Figure 8 displays the response of macro aggregates to negative shock to skewness of stock returns of one-standard deviation. Impulse responses are estimated using Local Projections (Jordà, 2005) using quarterly data fro 1974q1 to 2019q4. The Local Projection includes contemporaneous and lagged levels of the SP500, the P90-P10 spread of daily stock returns, GDP, Employment, Investment, Consumption, Fed Funds Rate, Hourly Wages, and CPI. All variables, with the exception of the Fed Funds Rate, the Kelley Skewness, and the P90-P10 differential, are in logs. We residualize Kelley Skewness from contemporaneous and lagged correlation with the same macro aggregates included in the Local Projection. The shaded areas represent 90% confidence intervals.

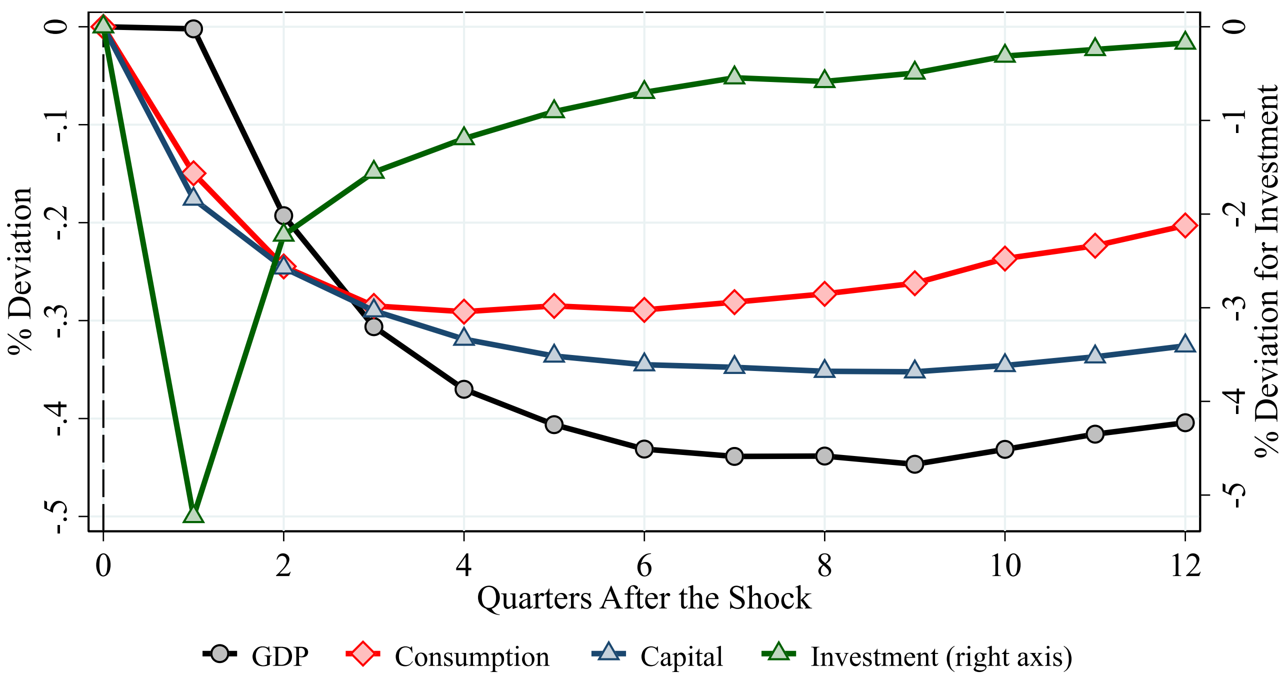
Figure 9 – Macro Outcomes Following Positive and Negative Skewness Shocks



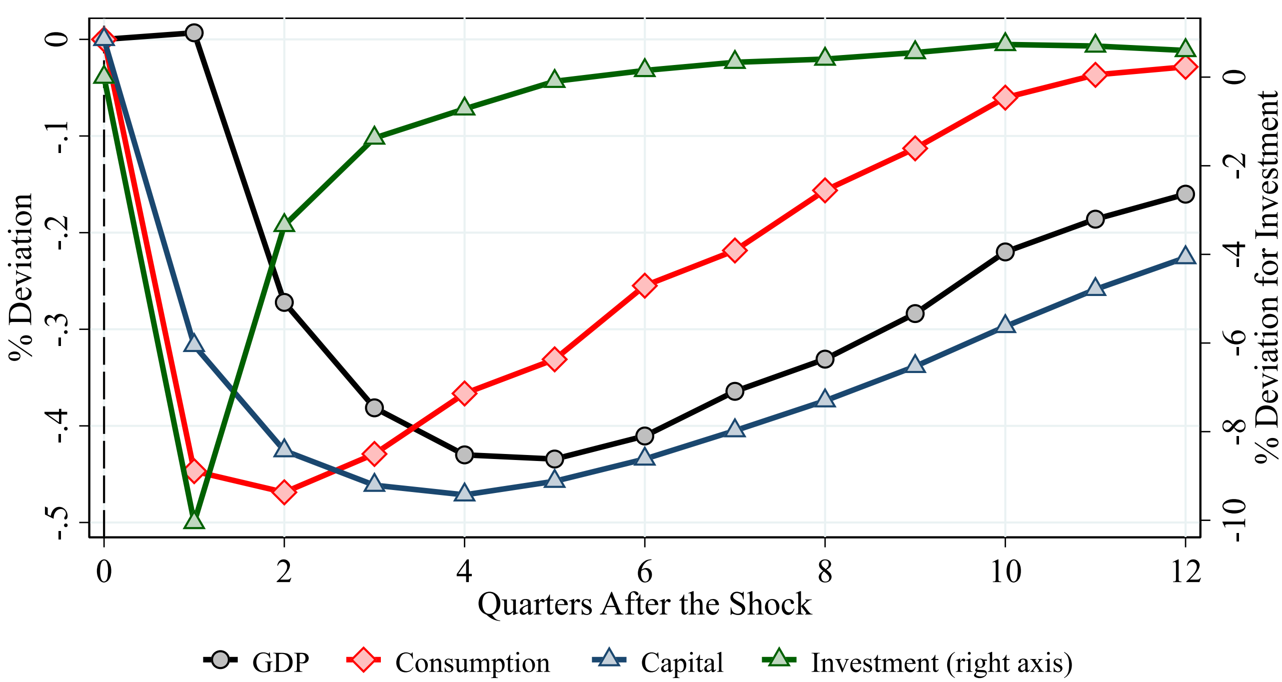

Note: Figure 9 displays the response of macro aggregates to a shock to skewness of stock returns of one-standard deviation for positive and negative changes in skewness. Impulse responses are estimated using Local Projections (Jordà, 2005) using quarterly data fro 1974q1 to 2019q4. The Local Projection includes contemporaneous and lagged levels of the SP500, the P90-P10 spread of daily stock returns, GDP, Employment, Investment, Consumption, Fed Funds Rate, Hourly Wages, and CPI. It also includes an interaction of the level of skewness and a dummy which is equal to 1 if skewness is positive (black line with circles) and a second interaction of the level of skewness and a dummy which is equal to one if skewness is negative (green line with squares). All variables, with the exception of the Fed Funds Rate, the Kelley Skewness, and the P90-P10 differential, are in logs. We residualize Kelley Skewness from contemporaneous and lagged correlation with the same macro aggregates included in the Local Projection. The shaded areas represent 90% confidence intervals.

Figure 10 – Macroeconomic Effect of a Shock to Skewness and Variance

(a) Skewness




(b) Variance and Skewness




Note: Figure 10 shows the effect of a decline in the skewness of firm idiosyncratic productivity. The plot is based on independent simulations of 1,000 economies of 300-quarter length. In each simulation, we assume that the economy is in the low-risk state for 150 periods. We then impose a drop in the skewness of firms' shocks in quarter 151, allowing the normal evolution of the economy afterwards. We plot the log percentage deviation of each macroeconomic aggregate from its value in quarter 0. The top panel shows the effects of output, whereas the bottom panel shows the impact on labor, investment in capital, consumption, and investment in the risk-free asset.

## Table I – The Skewness of Firm Growth is Procyclical

| | Kelley Skewness of Firms Growth | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | United States | | | | Cross-Industry US | | | | Cross-Country | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | $\text{Emp.}_t$ | $\text{Sales}_t$ | $\text{Sales}_t$ | $\text{Ret.}_t$ | $\text{Emp.}_{jt}$ | $\text{Sales}_{jt}$ | $\text{TFP}_{jt}$ | $\text{Ret.}_{jt}$ | $\text{Emp.}_{kt}$ | $\text{Sales}_{kt}$ | $\text{TFP}_{j(k)t}$ | $\text{Ret.}_{kt}$ |
| $\Delta GDP_{jt}$ | 0.062*** | 0.057*** | 0.084*** | 0.035** | | | | | 0.064*** | 0.020*** | | 0.022* |
| | (0.007) | (0.014) | (0.026) | (0.010) | | | | | (0.011) | (0.010) | | (0.011) |
| $\Delta Sales_{jt}$ | | | | | 0.088*** | 0.103** | 0.022*** | 0.020*** | | | 0.009*** | |
| | | | | | (0.020) | (0.042) | (0.010) | (0.004) | | | (0.002) | |
| $R^2$ | 0.61 | 0.50 | 0.15 | 0.12 | 0.15 | 0.18 | 0.26 | 0.39 | 0.31 | 0.37 | 0.29 | 0.44 |
| Obs. | 42 | 20 | 51 | 199 | 860 | 873 | 457 | 3,214 | 987 | 987 | 2,278 | 2,738 |
| Period | 76-18 | 98-17 | 70-20 | 70-20 | 70-20 | 70-20 | 76-19 | 70-20 | 84-17 | 84-17 | 99-18 | 85-20 |
| Freq. | Yr | Yr | Yr | Qtr | Yr | Yr | Yr | Qtr | Yr | Yr | Yr | Qtr |
| Source | LBD | LBD | CSTAT | CRSP | CSTAT | CSTAT | ASM | CSTAT | BvD | BvD | BvD | GCSTAT |
| Average | 0.02 | -0.03 | 0.09 | -0.09 | 0.09 | 0.06 | 0.00 | -0.09 | 0.12 | 0.05 | 0.02 | -0.09 |
| Expans. | 0.05 | 0.00 | 0.11 | -0.05 | 0.13 | 0.11 | 0.03 | -0.07 | 0.14 | 0.07 | 0.03 | -0.05 |
| Recess. | -0.10 | -0.15 | -0.07 | -0.12 | -0.04 | -0.13 | -0.03 | -0.15 | 0.02 | -0.04 | 0.01 | -0.13 |
| Sample | 4.6M | 4.6M | 4.5K | 4.0K | 4.5K | 4.0K | 30.0K | 4.0K | 12.3K | 21.0K | 411.0K | 38.0K |

Note: Columns (1) to (4) of Table I time-series regressions for the United States where the dependent variable is the Kelley skewness of the distribution of one-year firm employment and sales growth from the US Census' Longitudinal Business Database (LBD) (columns 1 and 2), one-year firm sales growth from Compustat (CSTAT) (column 3), and one-year stock returns from CRSP (column 4). In each regression, the independent variable is the one-year GDP per capita growth. Employment (Sales) growth is weighted by average employment (sales) of the firm between years $t$ and $t-1$. Newey-West standard errors are in parentheses below the point estimates. Regressions include a linear trend. Columns (5) to (8) show industry-panel regressions in which the dependent variable is the Kelley skewness of the within-industry distribution of firms-level outcomes using data from CSTAT (columns 5, 6, and 8) and the US Census' Annual Survey of Manufacturing (ASM). Industry $j$ is defined as a 2-digit NAICS; in column (7) an industry $j$ is a 4-digits NAICS cell within manufacturing (NACIS 31-33). In each regression, the independent variable is the average sales growth within an industry-year cell. Standard errors are clustered at the industry level. Compustat sample considers firms with 10+ years of data. All regressions include time and industry fixed effects. Columns (9) to (12) show country-level panel regressions in which the dependent variable is the Kelley skewness of different firm-level outcomes in country $k$ in period $t$. Employment and sales data come from BvD Osiris dataset, stock returns are from Global Compustat (GCSTAT), and TFP shocks are estimated using data from the BvD Amadeus dataset. In each regression, the independent variable is the one-year GDP per capita growth. Standard errors are clustered at the country level. In column (11) we further divide the sample within 2-digit NAICS. All regressions include year fixed effects and country fixed effects. Column 11 also includes industry fixed effects. Expansion is the average across expansion periods; Recession is the average across recession periods. In columns (1) to (4), Expansion is the weighted average across all years where the weight is the share of NBER non recession quarters within a year. Recession is defined analogously. In columns (5) to (12), an industry or country recession is defined as a year in which the average sales growth in the same sample is negative. Sample refers to the average firm-level observation per year. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table II – Firms' Growth is Lower When Skewness of Productivity Shocks is Lower

| Dependent Variable: | Sales Growth$_{i,t}^{j,k}$ | | | Employment Growth$_{i,t}^{j,k}$ | | | Investment$_{i,t}^{j,k}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $\mathcal{S}_{\mathcal{K}}{}_t^{j,k}$ | 0.29*** | 0.40*** | 0.27*** | 0.10*** | 0.11*** | 0.13*** | 0.14*** | 0.15*** | 0.08*** |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| $R^2$ | 0.00 | 0.31 | 0.30 | 0.00 | 0.28 | 0.32 | 0.00 | 0.29 | 0.30 |
| Obs. (M) | 10.97 | 10.97 | 6.95 | 9.53 | 9.53 | 6.95 | 10.3 | 10.3 | 6.68 |
| Year FE | N | Y | Y | N | Y | Y | N | Y | Y |
| Firm FE | N | Y | Y | N | Y | Y | N | Y | Y |
| Controls | N | N | Y | N | N | Y | N | N | Y |

Note: Table II shows a set of firm panel OLS regressions using firm-level data from BvD Amadeus. In all regressions, the independent variable is the Kelley skewness of firms' TFP shocks within an industry/country/year bin, denoted by subscripts $j, k$, and $t$ respectively. Firm-level dependent variables are the log change in firms' sales, the log change in firms' employment, and log change in firms' gross fixed assets. Controls include the median and standard deviation of TFP shocks within an industry/country/year bin, firm employment, a polynomial on firm age, the lag of the dependent variable and firm and year fixed effect. All regressions are weighted by firm employment. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

# Supplementary Online Appendix

# A Appendix: Robustness Results

Table A.1 – Data Sources and Sample Characteristics

| Variable | | Compustat | | Census | | BvD | BvD |
|---|---|---|---|---|---|---|---|
| | | | LBD | LBD-Rev | ASM/CM | Osiris | Amadeus |
| Employment | Mean | 9,364 | 23 | - | 89 | 6,257 | 25 |
| | P10 | 36 | 1 | - | 12 | 35 | 1 |
| | P50 | 1,078 | 4 | - | 70 | 631 | 4 |
| | P90 | 17,900 | 24 | - | 334 | 10,650 | 29 |
| | P99 | 145,000 | 177 | - | - | 108,256 | 302 |
| Sales (000s) | Mean | 2,912 | - | 5,059 | 1,100 | 1.08 | 9.14 |
| | P10 | 8 | - | 75 | 0.0 | 0.14 | 0.08 |
| | P50 | 325 | - | 426 | 6.05 | 0.72 | 0.44 |
| | P90 | 5,384 | - | 3,425 | 229 | 1.40 | 6.86 |
| | P99 | 49,686 | - | 36,940 | - | 16.90 | 9.89 |
| Frequency | | Annual | Annual | Annual | Annual | Annual | Annual |
| Period | | 1970-2020 | 1978-2019 | 1998-2018 | 1976-2018 | 1991-2015 | 1996-2018 |
| Obs. (M) | | 0.23 | 4.6 | 4.5 | 0.25 | 0.60 | 39.7 |
| Unit. of Obs. | | Firm | Firm | Firm | Estab. | Firm | Firm |
| Firm Type | | Pub. | Pub./Priv. | Pub./Priv. | Pub./Priv. | Pub. | Pub./Priv. |
| Countries | | US | US | US | US | Multiple | Multiple |
| Sectors | | All | Non Farm | Non Farm | Manuf. | All | All |

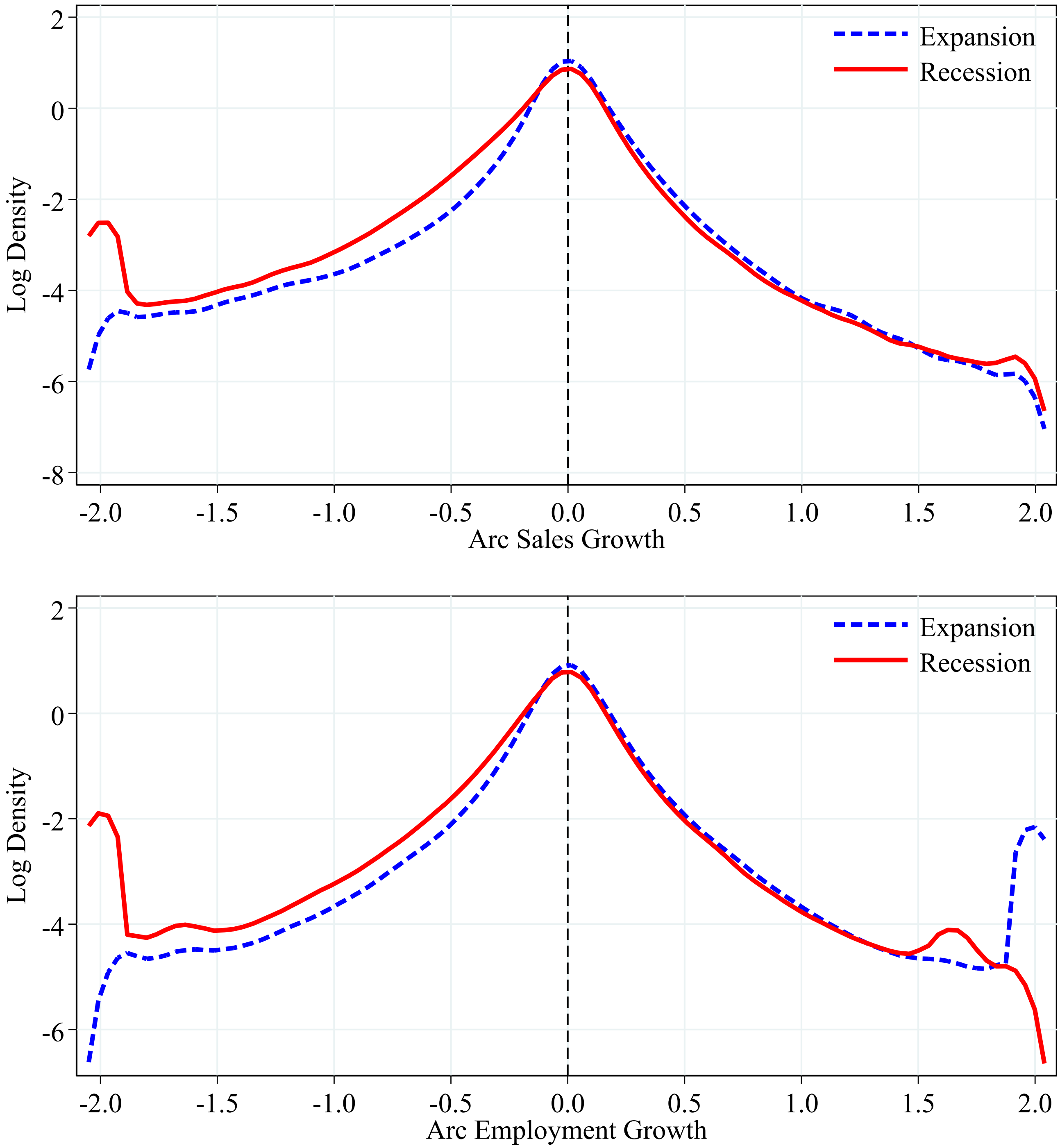
Note: Table A.1 shows the list of datasets and time-frames used in the analysis. Sample statistics correspond to 2010 for comparability. All monetary values are expressed in US dollars of 2010. We omit data from Global Compustat since it does not contain information on employment or sales. LBD sample statistics are aggregated at the firm-level. The 99th percentile is not reported to avoid disclosure of sensitive information. Total observations correspond to all sales observations across all years in the sample with valid observations of sales and employment. ASM results are calculated using sample weights. The 99th percentile of establishments sales and employment are not reported to avoid disclosure of sensitive information. See Table B.3 in the Appendix for a complete list.

## Table A.2 – Moments of the Skewness of Firm Growth

| | Kelley Skewness of Firm Growth | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | United States | | | | Cross-Industry US | | | | Cross-Country | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | Emp.$_{t}$ | Sales$_{t}$ | Sales$_{t}$ | Ret.$_{t}$ | Emp.$_{jt}$ | Sales$_{jt}$ | TFP$_{jt}$ | Ret.$_{jt}$ | Emp.$_{kt}$ | Sales$_{kt}$ | TFP$_{j(k)t}$ | Ret.$_{kt}$ |
| Mean | 0.02 | -0.03 | 0.09 | -0.09 | 0.09 | 0.06 | 0.00 | -0.09 | 0.11 | 0.05 | 0.02 | -0.07 |
| Std. Dev | 0.08 | 0.08 | 0.19 | 0.14 | 0.34 | 0.36 | - | 0.17 | 0.22 | 0.19 | 0.08 | 0.21 |
| Auto Corr. | 0.16 | 0.31 | 0.16 | 0.07 | 0.14 | 0.18 | - | 0.28 | 0.50 | 0.45 | 0.32 | 0.19 |
| P10 | -0.09 | -0.14 | -0.16 | -0.25 | -0.36 | -0.41 | - | -0.30 | -0.19 | -0.19 | -0.08 | -0.33 |
| P50 | 0.03 | 0.00 | 0.10 | -0.10 | 0.12 | 0.07 | - | -0.09 | 0.13 | 0.06 | 0.02 | -0.07 |
| P90 | 0.10 | 0.05 | 0.27 | 0.11 | 0.51 | 0.51 | - | 0.14 | 0.38 | 0.28 | 0.17 | 0.18 |
| Expansion | 0.05 | 0.00 | 0.11 | -0.05 | 0.13 | 0.11 | 0.03 | -0.07 | 0.14 | 0.07 | 0.03 | -0.06 |
| Rececess. | -0.10 | -0.15 | -0.07 | -0.12 | -0.04 | -0.13 | -0.03 | -0.15 | -0.02 | -0.04 | 0.01 | -0.14 |
| Source | LBD | LBD | CSTAT | CRSP | CSTAT | CSTAT | ASM | CSTAT | BvD | BvD | Amadeus | GC |
| Frequency | Yr | Yr | Yr | Qtr | Yr | Yr | Yr | Qtr | Yr | Yr | Yr | Qtr |
| Period | 76-18 | 98-17 | 70-20 | 70-20 | 70-20 | 70-20 | 76-15 | 70-20 | 84-18 | 84-18 | 99-18 | 85-20 |
| Obs. | 42 | 20 | 51 | 199 | 860 | 873 | 457 | 3,214 | 608 | 608 | 2,278 | 3,424 |
| Sample | 4.6M | 4.6M | 4.5K | 4.0K | 4.5K | 4.0K | 30.0K | 4.0K | 12.3K | 21.0K | 411.0K | 38.0K |

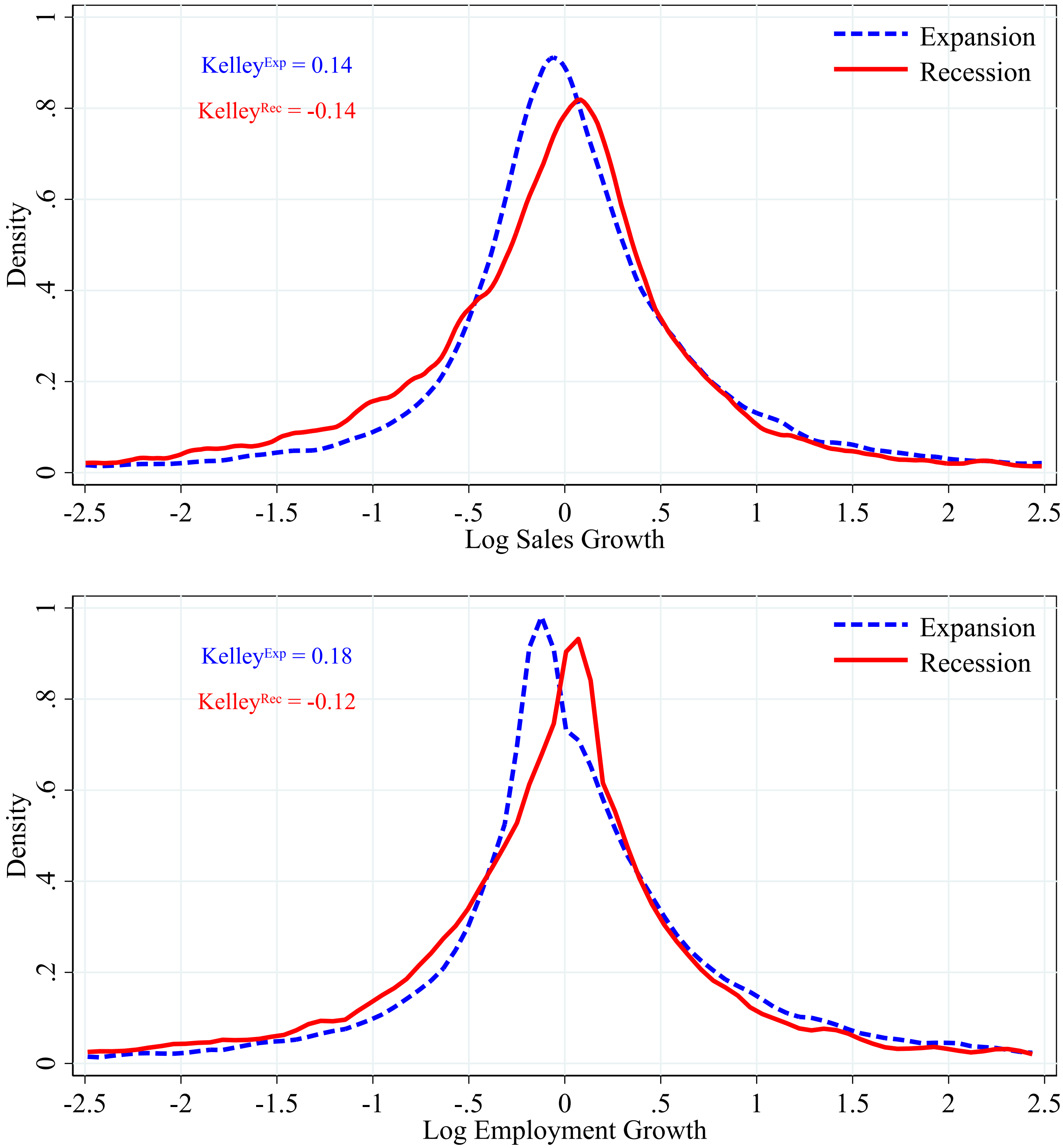
Note: Columns (1) to (4) of Table A.2 moments of the time-series of the Kelley Skewness of firm growth for the United States. Firm employment and sales growth is from the US Census' Longitudinal Business Database (LBD) (columns 1 and 2), firm sales growth from are Compustat (CSTAT) (column 4), and one-year stock returns from are CRSP (column 4). Columns (5) to (8) show industry-panel moments of the Kelley skewness of the within-industry distribution of firms-level outcomes using data from CSTAT (columns 5, 6), the US Census' Annual Survey of Manufacturing (column 7), and CRSP (column 8). Industry $j$ is defined as a 2-digit NAICS; in column (7) an industry $j$ is a 4-digits NAICS cell within manufacturing (NACIS 31-33). Columns (9) to (12) show country-panel moments of the Kelley skewness of the within-country distribution of firm-level outcomes using data from BvD Osiris dataset (columns 9 and 10 for 42 countries), BvD Amadeus (column 11 for 10 countries), Global Compustat (column 12 for 37 countries). Auto corr. is the one-year autocorrelation of the corresponding Kelley skewness. Mean Exp. is the average across expansion periods; Mean Rec. is the average across recession periods. In columns (1) to (4), the Mean Exp. is the weighted average across all years where the weight is the share of NBER non recession quarters within a year. Similarly, Mean Rec. is the weighted average considering the share of NBER recession quarters within a year. In columns (5) to (12), an industry or country recession is defined as a year in which the average sales growth in the same sample is negative. Obs. is the number of observations used to calculate the cross-sectional moments. Sample refers to the average firm-level observations per year.

Figure A.1 – Skewness of firm growth is pro-cyclical



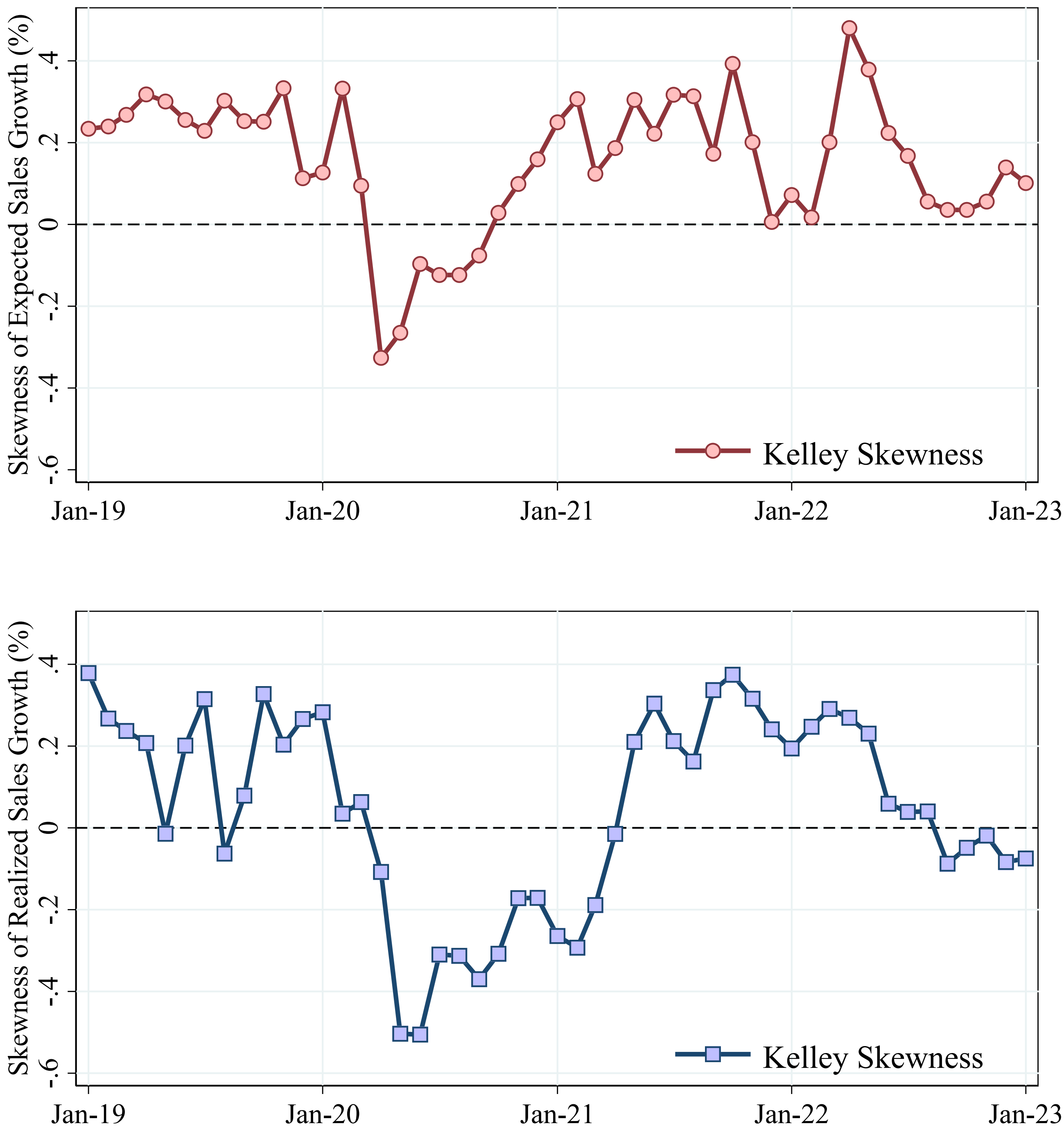

Note: Top panel (bottom panel) shows the sales-weighted (employment-weighted) log empirical density of the distribution of firms' arc-sales growth (arc-employment growth) between years $t$ and $t+1$ constructed from the US Census' Longitudinal Business Database (LBD) for firms (where firm employment sums across all establishments within the firm) with 20 employees or more. Arc-sales in period $t$ is calculated as $(x_{t+1} - x_t) / (0.5 \times (x_{t+1} - x_t))$. Each density has been adjusted to have a median of zero. The blue-dashed line shows the density of a pooled sample of expansion years (2003 to 2006 and 2010 to 2014) for a total of approximately 590,000 firm-year observations; the red-solid line shows the density of a pooled sample of recession years (2001 and 2008) for a total of approximately 570,000 firm-year observations.

Figure A.2 – Compustat: Skewness of firm growth is pro-cyclical



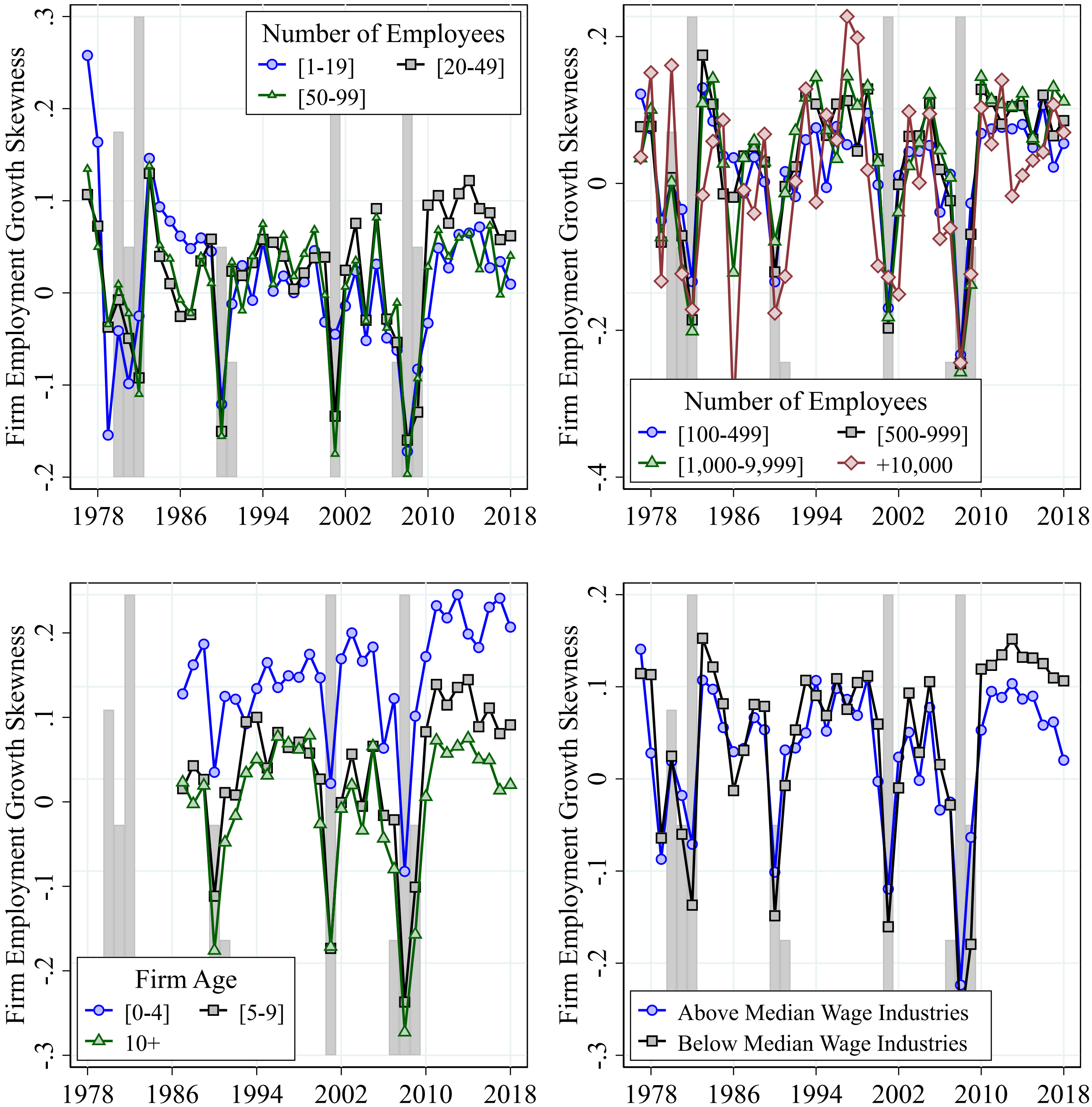

Note: The top panel (bottom panel) shows the sales-weighted (employment-weighted) empirical density of the distribution of firms' log sales growth and log employment growth between years $t$ and $t+1$ constructed from Compustat for a sample of firms with 10+ years of data. Each density has been adjusted to have a median of zero. The blue-dashed line shows the density of a pooled sample of expansion years (2003 to 2006 and 2010 to 2014) for a total of approximately 4,500 firm-year observations; the red-solid line shows the density of a pooled sample of recession years (2001 and 2008) for a total of approximately 4,500 firm-year observations.

Figure A.3 – COVID Skewness of Expected and Realized Sales Growth



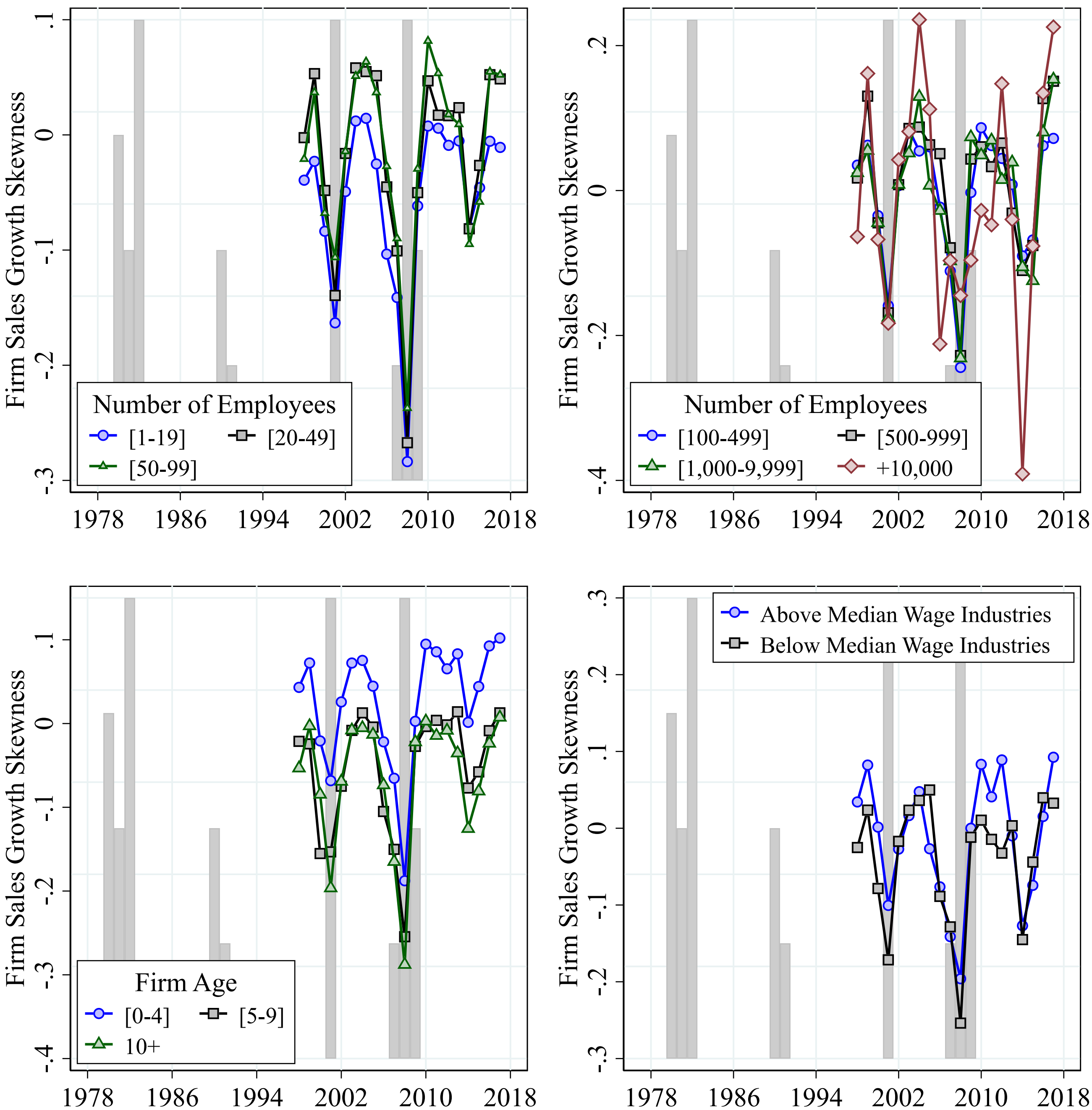

Note: Figure A.3 shows the Kelley Skewness of expected and realized annual sales growth for a representative sample of public and private firms in the United States. Data is collected by the Federal Reserve Bank of Atlanta with about 500 observations per month covering all major sectors and firm size groups. Kelley Skewness is calculated as $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] \, / \, (P90 - P10)$. Additional information on the methods and data collection can be found at www.atlantafed.org/research/surveys/business-uncertainty.

Figure A.4 – Skewness of Log Employment Growth is Procyclical by Firm Groups



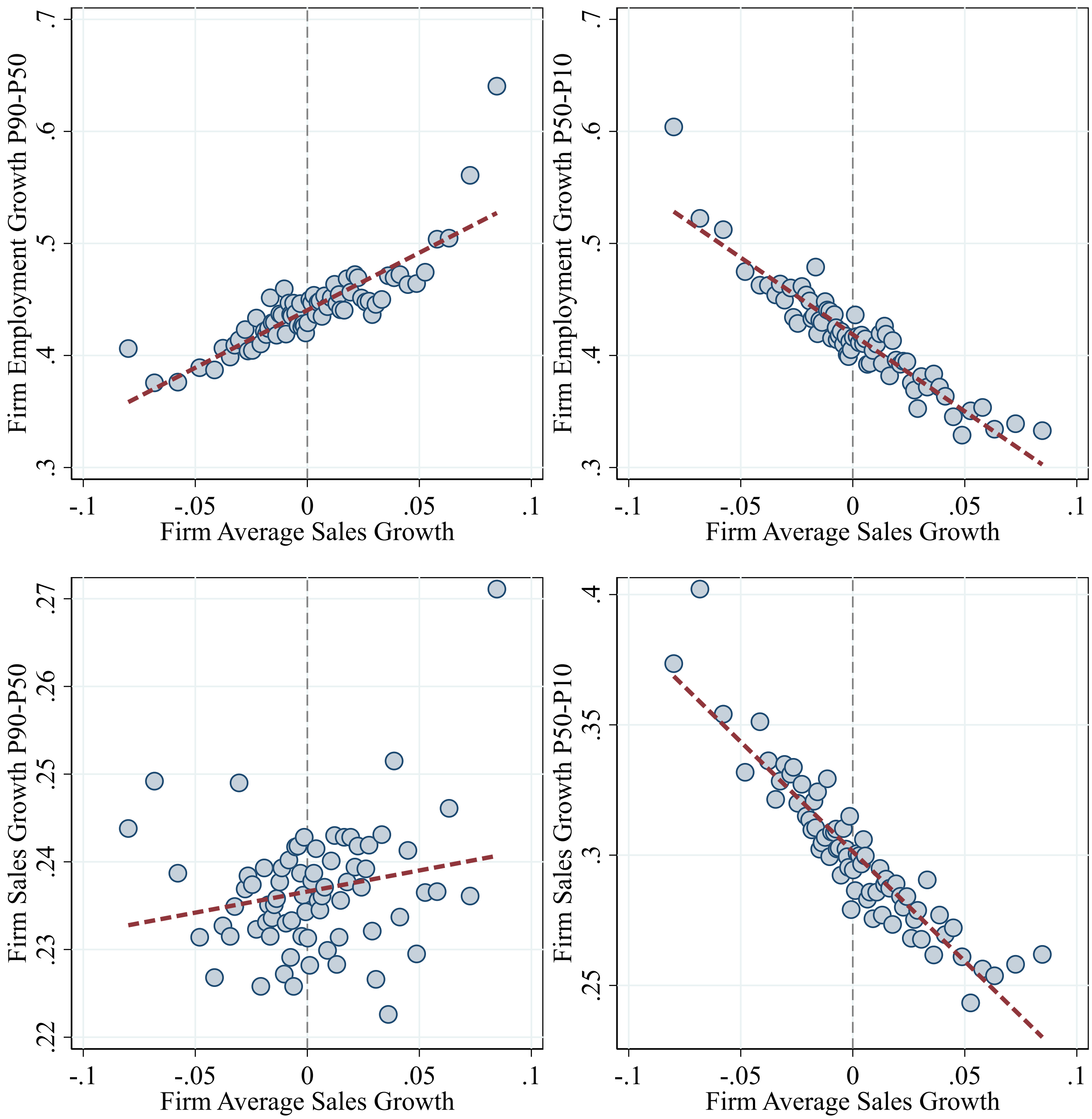

Note: Figure A.4 shows the time series of the cross-sectional Kelley skewness of the distribution of firm log employment growth for different firm groups using data from the US Census' Longitudinal Business Database (LBD), containing an average of about 4.6 million firm observations per year. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] / (P90 - P10)$ where $P90, P50, P10$ are employment-weighted percentiles of the employment growth distribution. Age groups start in 1987 since it is the first year in which we can identify firms with 10 or more years of age. Age for firms present in LBD in 1976 is not available. Number of employees is the total number of workers of a firm across all establishments. Shaded areas are the share of NBER recession quarters within a year.

Figure A.5 – Skewness of Log Sales Growth is Procyclical by Firm Groups




Note: Figure A.5 shows the time series of the cross-sectional Kelley skewness of the distribution of firm log sales growth for different firm groups using data from the US Census' Longitudinal Business Database (LBD), containing an average of about 4.6 million firm observations per year. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] / (P90 - P10)$ where $P90, P50, P10$ are sales-weighted percentiles of the sales growth distribution. Age for firms present in LBD in 1976 is not available. Number of employees is the total number of workers of a firm across all establishments. Shaded areas are the share of NBER recession quarters within a year.

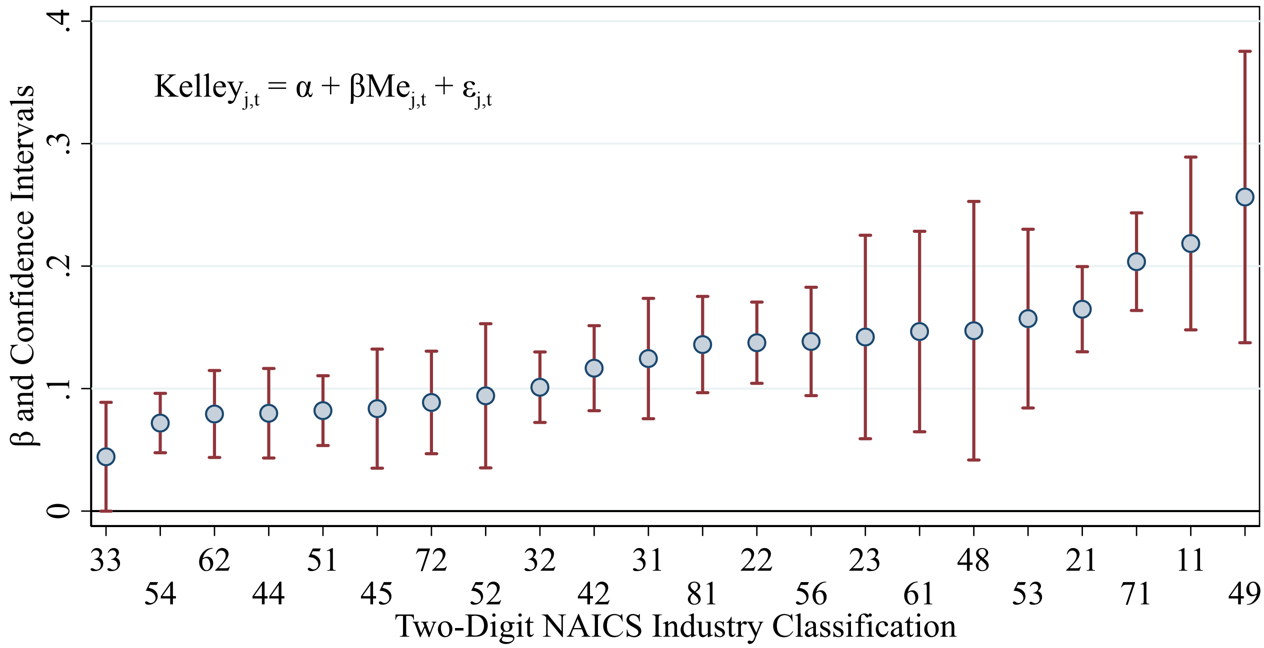
Figure A.6 – Right- and Left-Tail Dispersion and Industry Cycle



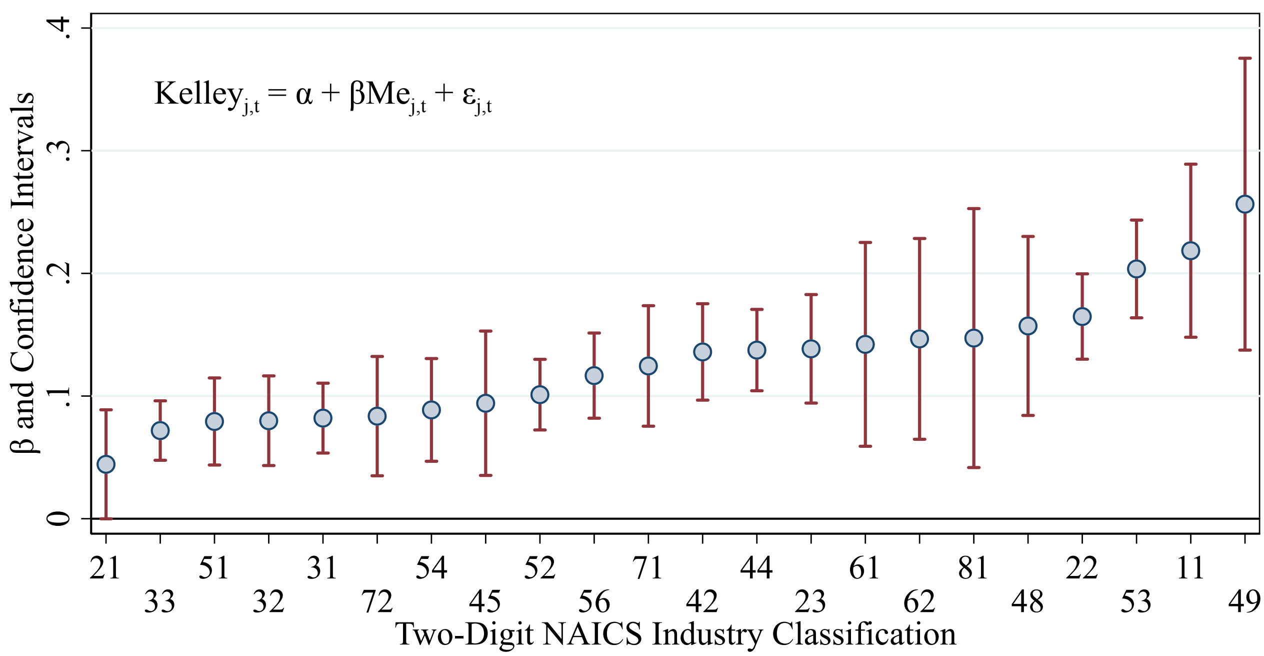

Note: The Figure A.6 shows bin-scatters of the P90-P50 and P50-P10 percentiles differential of firm employment growth (top panels) and firm sales growth (bottom panels) from the US Census' Longitudinal Business Database (LBD). Industries are defined as 4-digit NAICS for 284 industries covering the years 1998 to 2018 (sales data is only available from 1998 onwards in LBD), for an average sample of about 4.6 million firms per year. Bin-scatters control for industry and year fixed effects.

Figure A.7 – The Skewness of Firm Growth is Procyclical Within Industry

(a) Within-Industry Firm Employment Growth




(b) Sales Within-Industry Firm Sales GrowthGrowth




Note: Figure A.7 shows the coefficients and confidence intervals for within-industry regressions of the cross-sectional Kelley skewness on the average growth of employment (top panel) and sales (bottom panel) for a sample of publicly traded firms from Compustat. Each industry regression includes a linear trend. Confidence intervals are calculated at 95% of significance. Industries are defined as two-digit NAICS for 81 industries covering the years 1974 to 2019, for an average sample of about 4,500 firms per year. Kelley Skewness is $\mathcal{S}_{\mathcal{K}} = [(P90 - P50) - (P50 - P10)] \,/\, (P90 - P10)$ where $P90, P50, P10$ are percentiles of the corresponding distributions. In each plot, the dashed line is the coefficient of a panel regression of the within-industry skewness and average firm growth controlling for time and fixed effect.

# B Appendix: Data Description

This appendix describes the data sources, the sample selection, and the calculations of the moments we use for our empirical analysis. In Section B.2 we describe the firm- and establishment-level data for the United States obtained from the Census Bureau's Longitudinal Business Database (LBD). Section B.3 describes our sample of Compustat firms. For our cross-country comparisons, we use firm-level data available in the Bureau van Dijk's Osiris database and Global Compustat which we describe in Section B.4. Finally, Section B.5 describes our sample and TFP estimation for our sample of firms from the BvD Amadeus dataset and the establishment-level data for the US Census of Manufacturing and the Annual Survey of Manufacturing. The online appendix and replication packet—on the authors' websites—contains further details and corresponding do-files for most of our calculations.

## B.1 Firm-Level Survey Data

Firm-level information on expected and realized sales growth is obtained from the Survey of Business Uncertainty (SBU) developed by the Atlanta Fed in conjunction with Chicago Booth and Stanford. The goal of the survey is to elicit subjective probability distributions of from business executives and decisions makes about the macroeconomy and their own-firm outcomes such as sales, employment, or investment. The SBU began collecting data in 2014 and as of 2023, contains information from more than 2000 firms, across all 50 states in the United States, in every major non-farm sector, and across different segments of the firm-size distribution.

Every month, firm decision makers are asked, among other information, about their subjective expectations by means of a five-point subjective probability distribution over each firms' own four or twelve months future growth depending of the outcome variable (sales, employment, or capital expenditure). This flexible approach allows the responded to describe periods with different degree of growth, uncertainty, or more importantly for our purpose, if they expect positive or negative skewness of future outcomes. Importantly, since SBU also ask respondents about past realizations of the same variables, we are able to compare the skewness of expected and realized growth. Barrero (2022) documents additional properties of the subjective expectation probabilities and Altig *et al.* (2020) documents the significant changes observed in business uncertainty during the COVID pandemic. For our purpose, we can use the subjective probability across firms to construct percentiles of the *expected* sales growth distribution by weighting the expected sales growth with the support points probabilities provided by the responded. Additional details of the calculations can be found in Altig *et al.* (2022).

## B.2 United States: Longitudinal Business Database

We construct measures of employment growth at the firm-level using the Census Bureau's Longitudinal Business Database (LBD). The LBD covers the universe of establishment in the non-farm private sector in the United States from 1976 to 2019. It provides detailed establishment and firm-level information on employment, payroll, establishment location, firm age, industry, legal form of organization, and others. Crucially, firm and establishment identifiers in the LBD allow us to construct measures of employment growth at different time horizons. From the LBD, we select a sample of establishments that, in a given year, have nonnegative, non-missing employment and payroll and have valid industry data. We then sum up the employment within the same firm across all establishments to construct an annual measure of employment. Starting in 1998, the LBD also contains information on firm-level revenue (the so called LBDR). The coverage from the LBD to the LBDR is not complete. There is at least 30 percent of firms that have employment information (in LBD) but do not have revenue information (LBDR). A significant fraction of these firms (about 25%) are in the Other Services (except Public Administration) sector (NAICS 81). Importantly, the cross sectional moments for the employment growth distribution do not change when we focus in the sample of firms that have employment and revenue data. See Haltiwanger *et al.* (2016) for additional details on this data. We deflate nominal revenue measures to 2012 prices using the CPI (FRED series CPIAUCSL).

We measure the growth rate of employment of firm $j$ in period $t$ as the log-difference between periods $t$ and $t+1$, $g^e_{j,t} = \log E_{j,t+1} - \log E_{j,t}$. In order to capture the entry and exit of firms, we replace by a 0 the level of employment in the period before the first establishment is observed for the first time and the period after the last establishment of the firm is observed for the last time in the sample. We then calculate the growth rate of employment using the arc-percentage change between periods $t$ and $t+1$ which is given by $g^{arc}_{j,t} = \frac{E_{j,t+1}-E_{j,t}}{0.5\times(E_{j,t+1}+E_{j,t})}$. We take the arc-percent change as our baseline measure. We proceed in the same way for revenue growth.

To calculate the Kelley skewness we require the computation of specific percentiles of the distribution of employment growth. Notice that a percentile provides information about *one* particular firm, which violates the disclosure criteria imposed by the US Census Bureau. Hence, to avoid the disclosure of any sensitive information, we calculate the *pth* percentile of the employment growth distribution as the employment-weighted average on a band of +1 and -1 percent around percentile *pth*. For instance, the 90th percentile of the distribution is the weighted average of the employment growth across all observations between the 89th and 91st percentiles of the distribution, both ends included. We proceed in the same way to construct the 10th and 50th percentiles of the distribution and use these values to calculate the Kelley skewness, the 90th-to-10th log percentiles differential and the rest of the measures of dispersion. The massive sample size of the LBD (more than 4.6 million observations per year on average) ensures that the sample used to calculate each of the percentiles is large enough to have an accurate approximation to the actual quantiles of the distribution. All moments of the employment growth distribution are weighted by the average employment of the firm (or establishment in the case of establishment level results) between periods $t$ and $t+k$, that is, $\overline{E}_{j,t} = 0.5 \times (E_{j,t+k} + E_{i,t})$. Similarly, we weight revenue growth measures by the average real revenue between periods $t$ and $t+k$.

We also use the LBD to compare the empirical distribution of employment and revenue growth between recession and expansion years using a kernel density estimation. The sample selection is the same used in the rest of our results with a few exceptions. First, the densities are calculated using a pooled sample of firms for recession years (2001 and 2008) or expansion years (2003 to 2006 and 2010 to 2014 ). Second, we select a sample of firms that have at least 20 workers or more in year $t$. And third, the Census Bureau requires to drop the bottom and top 5% of the distribution when estimating empirical densities. The kernel densities presented in Figures 1 and 2 of the were calculated using the remaining sample.

## B.3 United States: Compustat

We construct time series of the cross-sectional dispersion and skewness of the sales growth distribution, the employment growth distribution, the stock returns distribution, and others using data of publicly traded firms from Compustat and CRSP accessed through the Wharton Research Data Services (WRDS).

For our results at the annual frequency, we obtain firm-level data from 1970 to 2020. The raw annual dataset contains 500,004 year/firm observations. We drop all observations with negative sales (Compustat variable sale), duplicated entries, and firms incorporated outside the United States (Compustat variable fic equal to "USA"). We also drop all observations that do not have a SIC classification or with a classification above 90. We deflate nominal variables using CPI (FRED series CPIAUCSL) and we calculate the growth rate of sales and employment (Compustat variable emp) as the log change between year $t$ and $t+k$ with $k \in \{1, 3, 5\}$. This leaves us with 266,192 firm/year observation (sales growth) between 1970 and 2019, with an average of 5,663 firms per year. Our main sample considers firms with at least 10 years of data (not necessarily continuous) but our results remain robust if we drop this restriction or if we consider firms with at least 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms, while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other).

To measure the degree of financially constraints, we use two measures i) by lagged average leverage and ii) by the lagged average interest coverage ratio. The first is calculated as the ratio between short and long term debt (sum of DLTT and DLC) and total value of assets. We then calculate the lagged level of leverage as the average value of the leverage between periods $t-1$ and $t-2$. Finally, we winsorise this measure at the value of 2. To calculate the interest coverage ratio we take EBIT divided by total interest expenses (TIE). If TIE is equal to 0, then the coverage ratio is set to a large number (coverage ratio of infinity). We then calculate the lagged level coverage ratio as the average value of the leverage between periods $t-1$ and $t-2$.

For our results based on quarterly data, we begin by retrieving firm-level data of net sales and stock prices at the annual and quarterly frequency, and employment at the annual frequency, from 1964q1 to 2020q4. The raw dataset of sales (Compustat variable saleq) and stock prices (Compustat variable prccq) contains more than 1.7 million quarter-firm observations with an average of approximately 4,660 firms per quarter. We drop all observations with negative sales, duplicated observations, and firms incorporated outside the United States (Compustat variable fic equal to "USA"). We also drop all observations that do not have a SIC classification or with a classification above 90. Then, we deflate nominal sales by the CPI (FRED series CPIAUCSL), and we calculate the growth rate of sales as the log-difference and the arc percentage change between quarter $t$ and $t+k$ with $k \in \{4, 12, 20\}$. This leaves us with around 1 million sales growth (log-difference) observations. For our main results, we consider firms with at least 10 years of data on quarterly sales (40 quarters, not necessarily continuous), which further reduces the sample to 819,977 observations between 1970q4 and 2017q2, with an average of 5,359 firms per quarter. Finally, in each quarter we calculate different cross-sectional moments discussed in the main body of this document. Our main sample considers firms with at least 10 years of data (40 quarters), although our results remain robust if we drop this restriction or if we consider firms with 25 years of data (more than 100 quarters). When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal to -2) under the assumption that before entering and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other).

## B.4 Cross-Country: BvD Osiris and Global Compustat

Cross-country firm-level panel data on sales and employment come from the Bureau van Dijk's Osiris database.[32] Osiris is a database of listed public companies, commodity-producing firms, banks, and insurance companies from over 190 countries. The combined industrial company dataset which we use in our analysis contains financial information for up to 20 years and 80,000 companies.

The raw dataset contains 977,412 country/firm/year observations from 1982 to 2018. We drop all observations with missing or negative sales, all duplicated entries, and all firms with missing NAICS classification. We transform all observations into US dollars using the exchange rate reported in the same database. Then, we deflate nominal sales using US annual CPI and calculate the growth rate of real sales as the log change and arc percentage change between years $t$ and $t+k$ with $k \in \{1, 3\}$. This leaves us with 748,574 observations (log change of sales). We further restrict the sample to firms with more than 10 years of data; country/year cells with more than 100 observations; countries with more than 10 years of data; and years with more than 5 countries. This sample selection reduces the dataset to an unbalanced panel of 678,563 observations in 45 countries between 1989 and 2015. We complement this data with real GDP in US dollars from the World Bank's World Development Indicators database.

The cross-country data on daily stock prices come from the Global Compustat database (GCSTAT), which provides standardized information on publicly traded firms for several countries at annual, quarterly, and daily frequencies. The raw data contain firm-level observations of daily stock prices between 1985 and 2018 for 48 countries. We drop all duplicated observations and drop all firms with less than 2000 observations (firms with approximately 10 years of data). Then we calculate daily price returns as the log-difference of the stock price between two consecutive trading days. We apply a similar sample selection, keeping firms with at least 10 years of daily price data. The total sample contains an unbalanced panel of 44 countries from 1985 to 2017 from which we drop all country quarters with less than 100 firms. The final data contains a total of 29 countries from 1985 to 2017. Then, within each quarter, we calculate the cross-sectional moments of the daily stock price distribution. We complement this dataset with per capita GDP growth from the World Bank's World Development Indicators and quarterly GDP growth from the OECD Stats. Table B.3 shows the list of countries available in our dataset and the data available for each country.

[32]See Kalemli-Ozcan *et al.* (2015) for additional details on the Orbis dataset.

TABLE B.3 – DATA AVAILABILITY BY COUNTRY

| Source: | BvD Osiris | | Global Compustat | BvD Amadeus | | | | BvD Osiris | | Global Compustat | BvD Amadeus | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sales | Emp | Returns | Sales | Emp | TFP | | Sales | Emp | Returns | Sales | Emp | TFP |
| ARG | x | x | | | | | IRN | x | x | | | | |
| AUS | x | x | x | | | | ISL | | | | x | x | |
| AUT | | | | x | x | | ISR | x | x | x | | | |
| BEL | x | x | x | x | x | | ITA | x | x | x | x | x | x |
| BLR | | | | x | x | | JPN | x | x | x | | | |
| BMU | x | x | | | | | KOR | x | x | x | | | |
| BRA | x | x | x | | | | MEX | x | x | | | | |
| CAN | x | x | | | | | MYS | x | x | | | | |
| CHE | x | x | x | x | x | | NLD | x | x | x | x | x | |
| CHL | x | x | x | | | | NOR | x | x | x | x | x | x |
| CHN | x | x | | | | | NZL | | | x | | | |
| DEU | x | x | x | x | x | x | PAK | x | x | | | | |
| DNK | x | x | x | x | x | | PER | x | x | | | | |
| EGY | x | x | | | | | PHL | x | x | | | | |
| ESP | x | x | x | x | x | x | POL | | | x | x | x | x |
| FIN | x | x | x | x | x | x | PRT | | | | x | x | x |
| FRA | x | x | x | x | x | x | RUS | x | x | x | | | |
| GBR | x | x | x | x | x | | SGP | x | x | | | | |
| GRC | x | x | x | x | x | | SWE | x | x | x | x | x | x |
| HKG | x | x | | | | | THA | x | x | | | | |
| HUN | | | | x | x | | TUR | x | x | x | | | |
| IDN | x | x | x | | | | UKR | | | | x | x | x |
| IND | x | x | x | | | | ZAF | x | x | x | | | |
| IRL | | | x | x | x | | | | | | | | |

Note: Table B.3 shows data available for each country (identified by its isocode). US data sources are omitted.

## B.5 TFP Estimation

### B.5.1 Cross-Country: BvD Amadeus

In this appendix, we describe in detail the construction of our measure of firm-level TFP using data from BvD Amadeus. This dataset has been used for several other researchers. For instance, Bloom and Van Reenen (2007) uses this dataset to evaluate management practices in the UK, French and German sampling frames. Bloom *et al.* (2012) discusses extensively the coverage of Amadeus. Some other paper have also calculated productivity using this data, more prominently Gopinath *et al.* (2017) in their analysis of misallocation in Spain and other European countries. We provide an extensive comparison of our data and results to those presented by these authors in subsection B.5.4. See also Kalemli-Ozcan *et al.* (2015) for details about constructing a representative dataset using BvD Amadeus data.

We consider a set of countries, namely, Germany, Spain, Finland, France, Hungary, Italy, Netherlands, Norway, Poland, Portugal, Sweden, and Ukraine, for which firm-level information is available for enough industries and sectors. For each country in the sample, we retrieve firm-level panel data from Amadeus trough WRDS. Our data contains a large range of firms, from small to very large firms (V+L+M+S: plus Small Companies dataset), both publicly traded and privately held. The main variables we use in our analysis are the following (Amadeus names of variables in parenthesis):

- Sales (TURN),
- Operating revenues (OPRE),
- Employment (EMPL),
- Cost of Employees (STAF),
- Cost of Material (MATE),
- Total Fixed Assets (FIAS),
- Industry (NAICS and SIC codes),
- Exchange rate from local currency to Euros (EXCHANGE2).

In order to estimate firm-level productivity for a large number of firms within each country, we perform a simple sample selection. For each country, we drop duplicates, observations without information on industry (NAICS), and firms with discrepancies between the country identifier and the firm identifier (INDR).[33] We also drop all observations with missing, zero, or negative values in either of the following variables: OPRE, MATE, FIAS, and STAF. We also drop all observation with zero or negative value of VA = OPRE − MATE which is our measure of value added. We deflate all monetary values by the country-specific CPI (obtained from the World Bank). All monetary values are transformed to Euros using the exchange rate supplied by BvD.

[33] The first two characters in the firm identifier in Amadeus refer to the country.

### B.5.2 Estimating TFP

The literature has considered several different methods to measure TFP at the firm-level (Syverson, 2011) and in this section we consider few standard methods. If we assume that the firm's production function is Cobb-Douglas, we can estimate the firm-level productivity, $z_{i,j,t}$, as the residual of the following equation,

$$\log y_{i,j,k,t} = \alpha_K \log K_{i,j,k,t} + \alpha_L \log E_{i,j,k,t} + z_{i,j,k,t}, \tag{3}$$

where $y_{i,j,k,t}$ is the value added of firm $i$, in industry $j$, in country $k$, in year $t$; $K_{i,j,k,t}$ is the deflated measure of fixed assets and $E_{i,j,k,t}$ is a measure of labor input (employees or wage bill).

We use four different methods to estimate $z_{i,j,t}$. The first method—which we use in our main empirical results—uses country-industry factor shares to estimate $\alpha_L$ and $\alpha_K$. In particular, we calculate the total wage bill and total value added at the country-industry-year level. Industries are defined by two-digit NAICS. To ensure our measure of factor shares is calculated with enough firms, we restrict our estimates to years in which the country-industry-cell contains more than one hundred observations and periods with more than five sectors within a country-year. We then obtain the labor share as

$$\alpha_{L,j,k,t} = \frac{\sum_{i \in I_{j,k,t}} w_{i,j,k,t}}{\sum_{i \in I_{j,k,t}} y_{i,j,k,t}},$$

where $I_{j,k,t}$ is the set of firms in the industry-sector-year cell and $w_{i,j,k,t}$ is the cost of employees at the firm-level (STAF). Then, we calculate the capital share as $\alpha_{K,j,k,t} = 1 - \alpha_{L,j,k,t}$.[34] We then apply these factor shares in equation (3) to obtain our first measure of productivity as the difference between $\log y_{i,j,k,t}$ and

$$\left( (1 - \alpha_{L,j,k,t}) \log K_{i,j,k,t} + \alpha_{L,j,k,t} \log E_{i,j,k,t} \right).$$

In the second method, we obtain $z_{i,j,t}$ as the residuals of a firm-level OLS panel regression. In order to control for differences in labor quality across firms, we use the wage bill (STAF) at the firm level as a measure of labor input. We then run, an OLS panel regression to obtain $\hat{z}_{i,j,k,t}$ for each firm.

The third approach—which is our baseline method—uses the methodology developed by Olley and Pakes (1996) to estimate $\hat{z}_{i,j,k,t}$. This method has stricter data requirements and therefore, we further restrict our within-country sample to firms with information about investment expenditure (change in the value of total fixed assets, FIAS), and firms with at least 5 years of data. Furthermore, because the data available in BvD Amadeus was increasingly populated until 2005, we consider information only after that year. To obtain the Olley and Pakes (1996) estimates we use the Stata command OPREG as implemented by Yasar *et al.* (2008).

The fourth method abstracts from capital differences across firms and proxies a measure of labor productivity. In particular, we obtain labor productivity as the residual of the following

[34] In this calculation, we use the nominal values of value added and cost of employees.

equation estimated using OLS within each country

$$\log y_{i,j,k,t} = \tilde{\alpha}_L \log E_{i,j,k,t} + \mu_i + \tilde{z}_{i,j,k,t}. \tag{4}$$

The fifth and six methods estimate productivity using Ackerberg *et al.* (2015) and the method of Wooldridge (2009) respectively, again using the Stata command OPREG.

Then, for each productivity measure, we estimate firm level productivity shocks as the residual of the following OLS panel regression within each country

$$\hat{z}_{i,j,k,t} = \beta_{0,k} + \beta_{1,k}\hat{z}_{i,j,k,t-1} + \mu_i + \delta_t + \epsilon_{i,j,k,t}, \tag{5}$$

where $\mu_i$ and $\delta_t$ are firm and year fixed effects respectively. In order to reduce the impact of outliers that normally appear in micro data, for each country, we winsorize each measure of productivity shock at the top and bottom 1%. Additionally, we use the average of the real sales growth within a bin (defined by country, industry, or year) as a measure of business conditions. Then, for each measure of productivity shock, we calculate the average shock within a country-industry-year bin and different percentiles of the distribution. To further ensure our results are not driven by outliers at the country-industry level, after we have obtained these percentiles, we trim the measures of Kelley skewness and the average productivity shocks at the top and bottom 1% and we restrict our sample to country-industry-year bins with more than 100 firms. Our results, however, follow through is we relax these conditions.

### B.5.3 Additional Evidence on the Skewness of Productivity Shocks

As we discussed in Section 3.3, the skewness of productivity shocks is robustly negative during periods of low economic activity within a country or an industry. Here we show some additional robustness results. Figure B.8 shows that the positive relation between the skewness of the productivity shocks and the business conditions is robustly positive, independently of the estimation method one uses to calculate firm-level productivity. For comparison, the top left panel repeats our main results shown in Figure B.8. Table B.4 shows a series of panel regressions in which the independent variable is the skewness of TFP shocks for each of the four methods described in Section B.5.2 and the main regressor is the average of the real sales growth (log change of operative revenues) within a country-industry-year cell. The coefficient associated to the average sales growth is positive and statistically significant at the 1% in all cases and of the same order of magnitude. This indicates that in periods in which industries experience a decline in sales, the skewness of the productivity shocks affecting the firms in that industry is negative as well.

Finally, Table B.5 shows that the skewness of firm's shocks is procyclical at the country level. In particular, we show the results of an industry-panel regression for each country in our sample using the average TFP shock as our measure of business condition. The results are indicative that the procyclicality of the skewness of firm shocks is not driven by any particular country in our sample and it is stronger in countries such as Germany and France.

### B.5.4 Comparing Sample with Gopinath *et al.* (2017)

High quality data is crucial to determine the business cycle variation of higher order moments of firms growth. more so when the goal is to analyze firm-level TFP. In our analysis, we use

Table B.4 – Positive Correlation of Sales Growth and Skewness of Firm's Shocks

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Estimation Method: | Factor Shares | Panel Regression | Olley and Pakes | Labor Productivity |
| Ave. Sales Growth | 1.21*** | 1.10*** | 1.18*** | 1.256*** |
| | (0.40) | (0.31) | (0.40) | (0.37) |
| $R^2$ | 0.18 | 0.14 | 0.31 | 0.17 |
| $N$ | 3,873 | 3,873 | 3,873 | 3,873 |

Note: Table B.4 shows a set of country-industry panel regressions in which the dependent variable is the Kelley skewness of firm productivity shocks calculated using the four different methods described in Section B.5.2. In all regressions, the explanatory variable is the average sales growth within the same bin. All regressions control for country, industry, and year fixed effects. Standard errors (below the point estimates) are clustered at the country level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

data from BvD Amadeus, which has been used before to calculate firm level productivity by other researchers. Hence, to further validate our analysis, we compare our sample to that of Gopinath, Kalemli-Özcan, Karabarbounis and Villegas-Sanchez (2017) (GKKV thereafter) that used the same data to study the effects of misallocation in Spain and other europeans countries using BvD Amadeus.

In what follows, we provide a step-by-step comparison of our analysis to that in GKKV. Performing this analysis was very useful to validate our construction, clean a few inconsistencies in the data, and improve in our estimation. The main difference between our results and theirs is that GKKV focuses mostly on manufacturing firms in Spain whereas we calculate productivity across all sectors available in our dataset. Having said that, we can follow their sample selection as close as possible and show, first, that the resulting cross-sectional statistics are qualitatively similar, and second, that our results (i.e., that the skewness of firms' productivity shocks is procyclical) are robust the sample restriction used by GKKV.

Beyond comparing our sample to GKKV, we perform a few additional robustness checks that further validate our analysis. In particular,

- We analyzed the relation between sector growth and the skewness of firms TFP shocks by firm characteristics (i.e., large and small firms and young and old firms)
- We further restricted our sample to firms with at least 5 years of data so as to control for the exit of firms that can affect the left tail of the firm growth distribution.
- Following the recommendation of Referee 3, we further orthogonalized firms productivity shocks from aggregate effects controlling for a flexible polynomial.

In all cases (showed in Appendix B.5 of the paper), we find our results to be robust to these changes. We decided, however, to keep our baseline data in the main text and relegate the comparison and results based on the GKKV sample selection to the Appendix B.5 of the paper. We would be happy to include these new results in the main text or expand in the comparison

Table B.5 – Skewness of Firms' Shocks is Procyclical at the Country Level

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| ISO | DEU | DNK | ESP | FIN | FRA | GBR | GRC | HUN | IRL |
| Sales | 5.27*** | 4.47*** | 1.69*** | 2.27*** | 3.24*** | 2.25*** | 1.27*** | 0.87*** | 1.70*** |
| Growth | (0.70) | (1.02) | (0.32) | (0.14) | (0.45) | (0.14) | (0.24) | (0.24) | (0.19) |
| $R^2$ | 0.66 | 0.63 | 0.60 | 0.64 | 0.56 | 0.74 | 0.47 | 0.76 | 0.66 |
| $N$ | 208 | 73 | 392 | 245 | 334 | 275 | 179 | 271 | 186 |

Continuation

| | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) |
|---|---|---|---|---|---|---|---|---|
| ISO | ISL | ITA | NLD | NOR | POL | PRT | SWE | UKR |
| Sales | 2.56*** | 1.29*** | 2.12*** | 2.04*** | 1.87*** | 1.74*** | 2.86*** | 1.56*** |
| Growth | (0.31) | (0.19) | (0.21) | (0.35) | (0.36) | (0.17) | (0.228) | (0.209) |
| $R^2$ | 0.84 | 0.55 | 0.72 | 0.51 | 0.52 | 0.61 | 0.65 | 0.55 |
| $N$ | 102 | 306 | 152 | 208 | 264 | 234 | 228 | 229 |

Note: Table B.5 shows a set of industry panel regressions in which the dependent variable is the Kelley skewness of firm productivity shocks. Firm-level productivity was calculated as the residuals of a firm-level panel regression (the second method described in section B.5.2). In each column, the independent variable is the average TFP shock within an industry. Each regression includes a set of industry and time fixed effects. Standard errors (below the point estimates) are clustered at the industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

between our sample and the results from GKKV if you think that is necessary. Below, we expand on these results in more detail. Thanks again for pushing us in this direction.

**Sample Selection.** Following GKKV, we clean our data in five steps within each country. First, we control for potential reporting errors. Second, we verify the internal consistency of the data we use in the analysis. Third, we implement additional controls on the specific variables we use in the TFP estimation step. Fourth, we winsorize the variables used in the analysis. We then drop duplicated observations in the dataset. We compare the resulting sample statistics with that presented in the published version of GKKV and their online appendix. Finally, we use the resulting dataset to calculate productivity using the same methods we use in our baseline analysis to show that our results are robust the sample selection used by GKKV. In what follows, we list this cleaning in detail following the same order they are applied to the data. The variable names available in BvD Amadeus are provided in parenthesis when appropriate.

- Step 1: Correcting reporting mistakes
  - We drop year-firm observations with missing in all the following variables at the same time: total value of assets (FIAS), operating revenues (OPRE), sales (TURN), and employment (EMP)

- We drop firms with negative value of FIAS in any year; firms with negative EMP or EMP above 2 million workers in any year; firms with negative TURN in any year; firms with negative value of fixed assets (TFAS) in any year.
- We drop firm-year observations with missing value of materials (MATE) or non-positive value of MATE; firm year observations with missing value of OPRE or non-positive value of OPRE; firm year observations with missing value of FIAS or non-positive value of FIAS

• Step 2: Internal consistency

- We drop firm-year observations with a ratio of the sum of TFAS, intangible assets (IFAS), and other assets (OFAS) to FIAS greater than 1.

• Step 3: Variable specific controls

- We drop firms-year with negative value of age, where age is constructed as the calendar year minus the date of incorporation of the firm plus one.
- We drop firm-year observations with missing value of wage bill (STAF) or non-positive value of STAF; Firm-year observations with negative value of IFAS; Firm-year observations with a ratio of TFAS to IFAS greater than 1; Firm-year observations with TFAS plus IFAS equal to 0.
- We drop firm-year observation with missing industry classification and with NAICS in the public sector (Four digit NAICS above 9000).
- We drop firm-year observations in the top and bottom 0.1% of the capital stock-to-wage bill distribution, where capital stock is defined as TFAS plus IFAS.
- We drop firm-year observations with negative value added (VA) where VA is calculated as OPRE minus MATE. We then drop firm-year observations with STAF-to-VA ratio at the top and bottom 0.1% of the distribution or with STAF-to-VA ratio above 1.1.

• Step 4: Winsorization

- We winsorize OPRE, STAF, TURN, and VA at the bottom 1% and to 0.5%.

• Step 5: Repeated variables

- Some firm-year records are repeated in the sample, we drop these repeated firm-year observations.

We save the resulting database for each country.

**Calculating Productivity.** Using this new dataset and our baseline dataset, we recalculate firm-level TFP and evaluate whether the skewness of firm-level productivity shocks remains procyclical. In particular, as we further discuss Appendix B.5, in the new version of the draft we include new results based on six different productivity estimation methods: factor shares, revenue per worker, the method of Olley and Pakes (1996), the method of Ackerberg *et al.* (2015), and the one proposed by Wooldridge (2009).

To calculate productivity, we apply each of the previous methods and we obtain a measure of productivity level $z_{jt}^k$ for firm $j$ in country $k$ in period $t$. We then regress within each country

$$\log z_{jt}^k = \rho z_{jt-1}^k + \lambda_j + \delta_t + \epsilon_{jt}^k,$$

where $\lambda_j$ and $\delta_t$ are a firm and a year fixed effect. Our measure of shocks is then given by $\hat{\epsilon}_{jt}^k$. To further control for outliers we,

- Step 1: Trim $\hat{\epsilon}_{jt}^k$ at the top and bottom 0.1%.
- Step 2: Calculate cross-sectional moments of the firm sales growth and TFP shock distribution only for year-country-sector bins with more than 100 firms.
- Step 3: Winsorize the distribution of mean sales growth and Kelley Skewness at the top and bottom 5%.

None of these changes have major consequences in our results. We use the resulting data to evaluate the relation between firm growth and skewness. The results based on the baseline built of the data are shown in Figure B.8 whereas the results based applying the sample selection of GKKV are shown in Figure B.9. The two main take aways from comparing these figures is, first, that in all cases, we find that the skewness of firm-level productivity shocks is procyclical, and second, that our results are robust to applying the sample selection of GKKV.

**Database Comparison and Results.** Finally, we compare different cross-sectional moments available in GKKV and their appendix, to similar cross-sectional moments coming from our dataset set under the GKKV sample selection. This serve as a further validation that our dataset is suitable for the analysis we perform. Across our analysis, we compare our data to the aggregates reported by Eurostats through its Structural Business Statistics database (SBS). In particular, we obtain information for aggregate level employment (SBS table tin00148), aggregate level gross output (SBS table tin00146), sector level wages (SBS table tin00154), sector level employment (SBS table tin00151), and sector level gross output (tin00149). All variables are expressed in Euros of corresponding year.

First, we compare the share of employment, wage bill, and gross output in manufacturing available in GKKV to the results coming from our dataset for the years in which our data overlaps with in GKKV's sample. This is shown in Table B.6. Relative GKKV, our sample on manufacturing covers a smaller fraction of the wage bill and gross output. Coverage of employment, however, is closer although still below that of GKKV. This is to be expected since we only use the data available through WRDS whereas GKKV use data of different vintages and discs that increase the their sample size. When we consider the whole economy, that is, including all firms and sectors after sample cleaning, the coverage of our dataset is still significant: across different variables and years, our sample accounts for between 40% and 54% of aggregate economic activity in Spain. This is also true across the rest of the countries we use in our dataset, as shown in Table B.7.

We then calculate cross-sectional moments of the distribution of five key variables: value added, employment, wage bill, total value of assets, and sales. The results in our data are shown in Table B.8 for the whole economy and manufacturing. Relative to GKKV, that show similar results in their online appendix, our firms are smaller across different dimensions, but qualitatively, the cross sectional moments are comparable. We conclude that, despite difference, our data captures a significant fraction of economic activity across the countries in our sample, and therefore, can be used to draw conclusions about the relation between aggregate economic activity and firms' growth

Table B.6 – Share of Employment, Wage Bill, and Gross Output Available in Sample

| | Gopinath et al. 2017 Spain Manufacturing | | | Salgado et al. (2023) Spain Manufacturing | | | Spain Economy | | |
|---|---|---|---|---|---|---|---|---|---|
| | Emp. | Wage Bill | Gross Output | Emp. | Wage Bill | Gross Output | Emp. | Wage Bill | Gross Output |
| 2009 | 0.71 | 0.72 | 0.75 | 0.54 | 0.44 | 0.39 | 0.45 | 0.43 | N/A |
| 2010 | 0.68 | 0.73 | 0.74 | 0.56 | 0.45 | 0.40 | 0.48 | 0.38 | 0.40 |
| 2011 | 0.69 | 0.74 | 0.75 | 0.60 | 0.48 | 0.40 | 0.52 | 0.40 | 0.41 |
| 2012 | 0.65 | 0.71 | 0.72 | 0.63 | 0.50 | 0.41 | 0.54 | 0.42 | 0.42 |

Note: This table shows the share of employment, wage bill, and gross output available in the dataset of manufacturing firms in GKKV (left panel), in our dataset (center panel), and across the entire economy (right panel) for Spain. Firm level data coming from BvD Amadeus database whereas country- and industry-level data from Structural Business Statistics database. The center panel uses a total sample of 632,948 firm-years observations. The right panel uses a total sample of 3,947,773 firm-years observations across all years available.

Table B.7 – Share of Employment, Wage Bill, and Gross Output Available in Sample

| | Year 2012 | | | Year 2016 | | |
|---|---|---|---|---|---|---|
| Country | Emp. | Wage Bill | Gross Output | Emp. | Wage Bill | Gross Output |
| Germany | 0.36 | 0.37 | 0.36 | 0.34 | 0.34 | 0.34 |
| Spain | 0.54 | 0.42 | 0.42 | 0.65 | 0.65 | 0.65 |
| Finland | 0.53 | 0.46 | 0.38 | 0.57 | 0.57 | 0.57 |
| France | 0.36 | 0.36 | 0.36 | 0.44 | 0.44 | 0.44 |
| Hungary | 0.40 | 0.48 | 0.50 | 0.40 | 0.40 | 0.40 |
| Italy | 0.48 | 0.41 | 0.45 | 0.59 | 0.59 | 0.59 |
| Norway | N/A | 0.69 | 0.49 | 0.96 | 0.96 | 0.96 |
| Poland | 0.12 | 0.31 | 0.33 | 0.13 | 0.13 | 0.13 |
| Portugal | 0.57 | 0.48 | 0.50 | 0.71 | 0.71 | 0.71 |
| Sweden | 0.37 | 0.26 | 0.26 | 0.47 | 0.47 | 0.47 |

Note: This table shows the share of employment, wage bill, and gross output available in the dataset relative to the economy-wide levels for two years across all countries in the Amadeus sample. Firm level data coming from BvD Amadeus database whereas country- and industry-level data from Structural Business Statistics database.

Table B.8 – Cross-Sectional Statistics

| | | Economy Wide | | Manufacturing | | | | Economy Wide | | Manufacturing | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Mean | Std. Dev. | Mean | Std. Dev. | | | Mean | Std. Dev. | Mean | Std. Dev. |
| Spain | Value Added | 0.8 | 3.2 | 1.3 | 4.2 | Hungary | Value Added | 4.5 | 4365.7 | 6.8 | 5647.1 |
| | Employment | 19.1 | 373.9 | 20.9 | 173.3 | | Employment | 91.9 | 402.7 | 168.9 | 457.1 |
| | Wage Bill | 0.4 | 1.3 | 0.6 | 1.6 | | Wage Bill | 1.1 | 983.5 | 2.0 | 1321.4 |
| | Total Assets | 2.2 | 78.9 | 2.3 | 41.8 | | Total Assets | 6.3 | 27922.2 | 10.9 | 44051.1 |
| | Sales | 2.0 | 8.2 | 3.1 | 10.9 | | Sales | 10.3 | 11415.6 | 17.6 | 15712.2 |
| Italy | Value Added | 1.6 | 5.9 | 2.4 | 6.8 | Poland | Value Added | 2.2 | 27.6 | 3.1 | 35.2 |
| | Employment | 19.9 | 239.1 | 25.6 | 153.2 | | Employment | 82.6 | 412.8 | 127.7 | 307.6 |
| | Wage Bill | 0.5 | 1.7 | 0.8 | 2.1 | | Wage Bill | 0.6 | 6.9 | 0.9 | 8.3 |
| | Total Assets | 2.2 | 98.9 | 2.3 | 43.2 | | Total Assets | 2.9 | 120.6 | 3.8 | 97.4 |
| | Sales | 3.0 | 12.0 | 4.8 | 14.5 | | Sales | 5.1 | 66.0 | 8.2 | 92.0 |
| Portugal | Value Added | 0.5 | 2.0 | 0.8 | 2.4 | Sweden | Value Added | 1.0 | 35.2 | 1.4 | 41.9 |
| | Employment | 13.6 | 136.5 | 19.9 | 62.1 | | Employment | 13.0 | 148.2 | 15.4 | 66.3 |
| | Wage Bill | 0.2 | 0.6 | 0.3 | 0.7 | | Wage Bill | 0.5 | 14.7 | 0.7 | 17.3 |
| | Total Assets | 1.3 | 58.7 | 0.9 | 8.5 | | Total Assets | 1.5 | 497.6 | 1.4 | 206.2 |
| | Sales | 1.1 | 4.4 | 1.6 | 5.4 | | Sales | 2.2 | 83.8 | 2.9 | 98.8 |
| Germany | Value Added | 39.3 | 116.1 | 37.8 | 106.8 | Ukraine | Value Added | 43.7 | 106.6 | 42.6 | 112.8 |
| | Employment | 499.6 | 3440.3 | 415.8 | 3217.1 | | Employment | 200.4 | 1564.1 | 257.5 | 967.8 |
| | Wage Bill | 14.3 | 39.7 | 14.4 | 37.7 | | Wage Bill | 6.0 | 12.2 | 8.0 | 15.7 |
| | Total Assets | 53.7 | 732.7 | 61.8 | 957.2 | | Total Assets | 71.0 | 1350.9 | 58.9 | 540.8 |
| | Sales | 81.9 | 256.8 | 78.7 | 231.0 | | Sales | 56.7 | 128.6 | 74.0 | 163.2 |
| France | Value Added | 2.1 | 8.0 | 3.9 | 11.6 | Finland | Value Added | 1.3 | 6.1 | 2.3 | 9.0 |
| | Employment | 41.3 | 1095.4 | 58.0 | 555.7 | | Employment | 22.8 | 249.6 | 38.3 | 355.9 |
| | Wage Bill | 0.8 | 2.8 | 1.5 | 4.0 | | Wage Bill | 0.5 | 2.2 | 0.9 | 3.2 |
| | Total Assets | 3.8 | 246.5 | 4.7 | 104.8 | | Total Assets | 1.5 | 71.3 | 2.4 | 32.4 |
| | Sales | 4.0 | 15.3 | 7.2 | 22.3 | | Sales | 2.6 | 13.3 | 4.6 | 19.2 |
| Norway | Value Added | 2.1 | 71.8 | 3.9 | 107.0 | | Value Added | 97.8 | 158.7 | 65.3 | 77.6 |
| | Employment | 11.1 | 78.1 | 16.2 | 60.4 | | Employment | 1307.0 | 2534.0 | 1296.3 | 2155.4 |
| | Wage Bill | 1.0 | 29.9 | 1.8 | 44.0 | | Wage Bill | 38.2 | 58.8 | 19.8 | 24.0 |
| | Total Assets | 4.4 | 1013.9 | 8.6 | 1522.7 | | Total Assets | 95.4 | 213.4 | 125.5 | 274.1 |
| | Sales | 4.5 | 160.9 | 8.6 | 244.5 | | Sales | 313.1 | 451.3 | 217.1 | 253.0 |

Note: This table shows cross-sectional moments of the distribution of key firm-level variables used in the calculation of firm-level productivity. All values are expressed in millions of euros. We deflate nominal values by country-level CPI. Moments are first calculated within each year and then averaged across all years within a country. Firm level data coming from BvD Amadeus database.

### B.5.5 Controlling for non-linear responses

Although our results robustly suggest that firm skewness is procyclical, it is possible that this relation is explained by non linear responses of firms's TFP to aggregate shocks that are not week capture by fixed average differences across years—i.e., they are not stripped out by the time fixed effect in Equation 5). One way to evaluate whether firms revenue TFP is differentially exposed to aggregate fluctuations is to strip out the covariance with respect to aggregate GDP. As a reminder, in our baseline results, we measure TFP shocks as the residuals of the following panel regression, which we run for each country in our sample,

$$\log z_{i,t} = \rho \log z_{i,t-1} + \lambda_i + \delta_t + \epsilon_{i,t}. \tag{6}$$

We can capture potential non-linear responses to a common aggregate shock by interacting $\log z_{i,t-1}$ with with a linear, quadratic, or cubic function of GDP growth, that is, for a quadratic case we estimate for each country

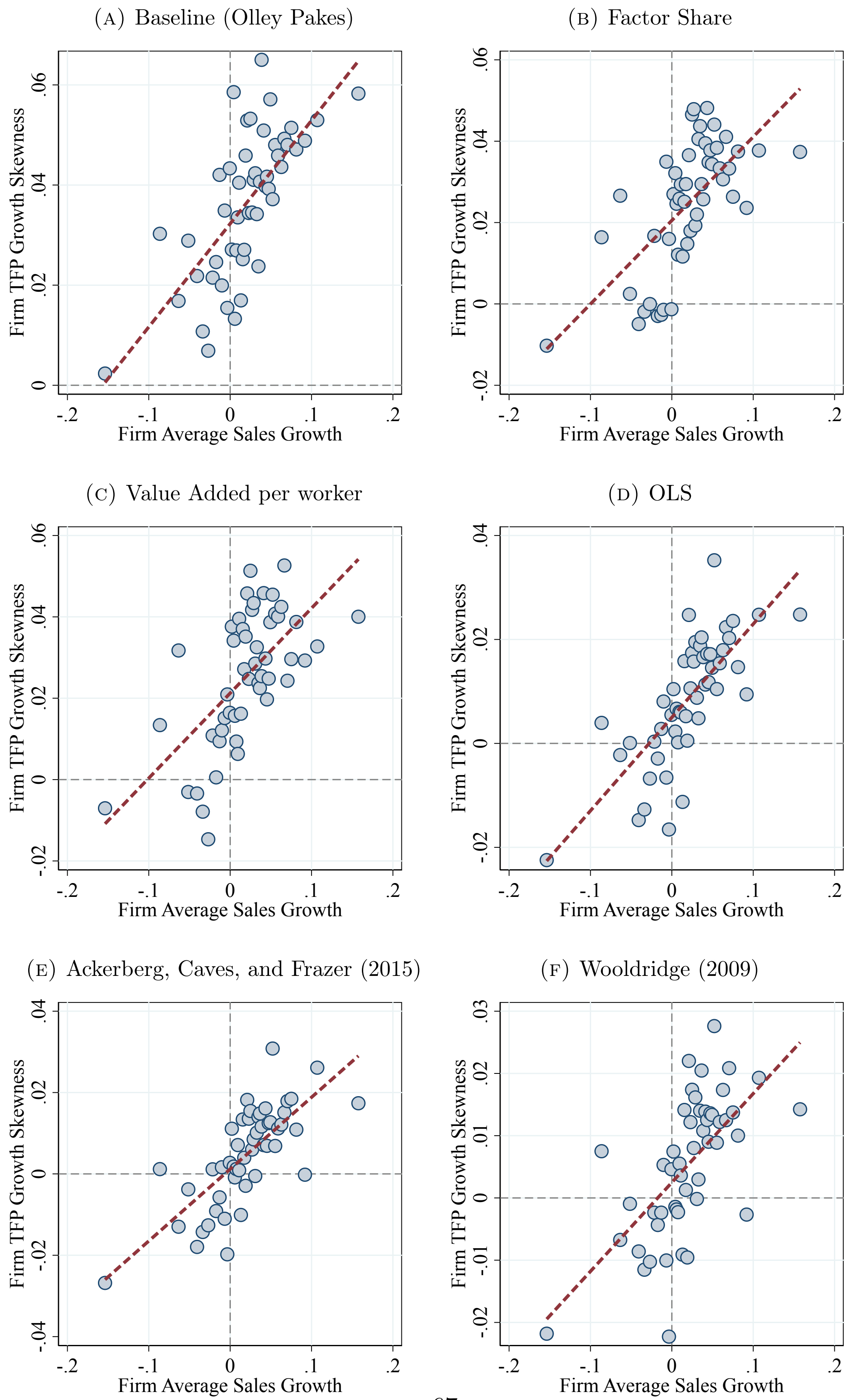
$$\log z_{i,t} \quad = \quad \rho \log z_{i,t-1} + \beta_1 \log z_{i,t-1} \times \Delta GDP_t + \beta_2 \log z_{i,t-1} \times \Delta GDP_t^2 + \delta_t + \lambda_i + \eta_{i,t}. \tag{7}$$

Alternative, it is possible that firms are differentially exposed to the business cycle so that $\beta_1$ for instance is sector or firm specific. To evaluate this case, we can estimate the following specification

$$\log z_{i,t} = \beta_{i,0} + \rho \log z_{i,t-1} + \beta_{i,1} \times \Delta GDP_t + \nu_{i,t}, \tag{8}$$

and then study whether different estimates of $\eta_{i,t}$ or $\nu_{i,t}$, change the cyclicality in the skewness that we observe in our baseline specification. We implement specifications 7 and 8 using our baseline TFP estimation results—which are based on the standard Olley and Pakes (1996) methods—using data from BvD Amadeus. Results based on US Census ASM are similar but not disclosed. The results, shown in Figure B.10, indicate that the skewness of TFP residuals is procyclical. We conclude that controlling for potential non linear responses might affect our results, the baseline conclusion remains unaltered.

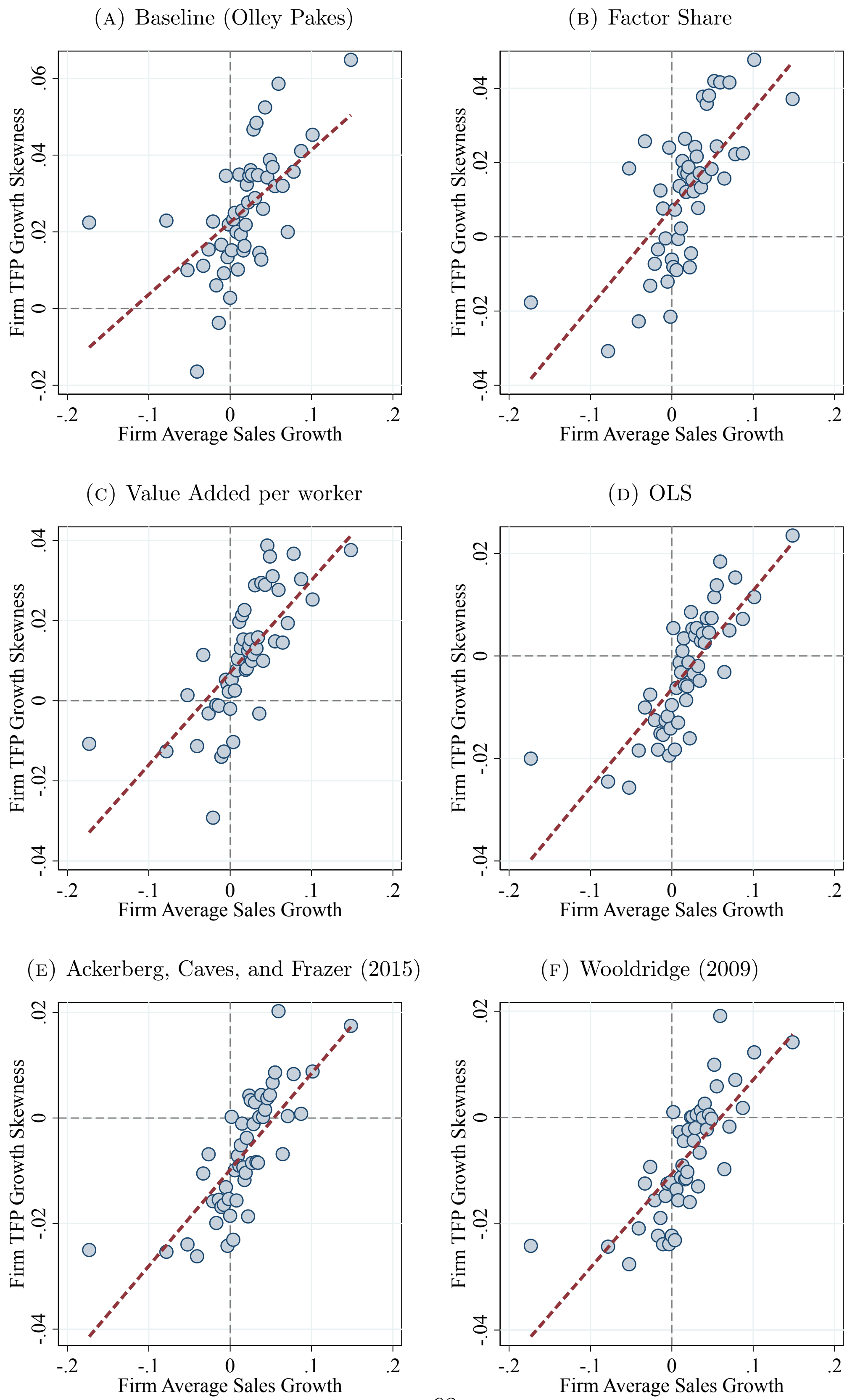
Figure B.8 – Baseline: Skewness of TFP shocks is Procyclical






Note: This figure shows the relation between the average firm sales growth within a year-country-industry and the skewness of firm productivity shocks within the same bins calculated using different methods. Binscatters control for year, country, and industry fixed effects. Industries defined as two digit NAICS.

Figure B.9 – GKKV: Skewness of TFP shocks is Procyclical

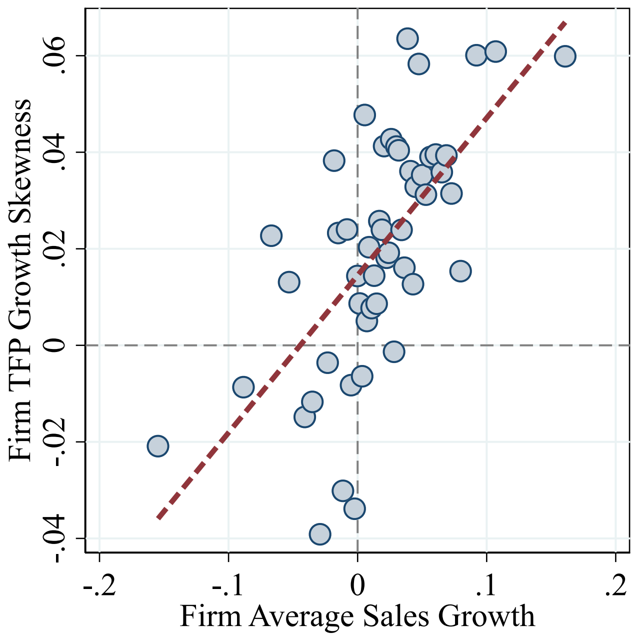




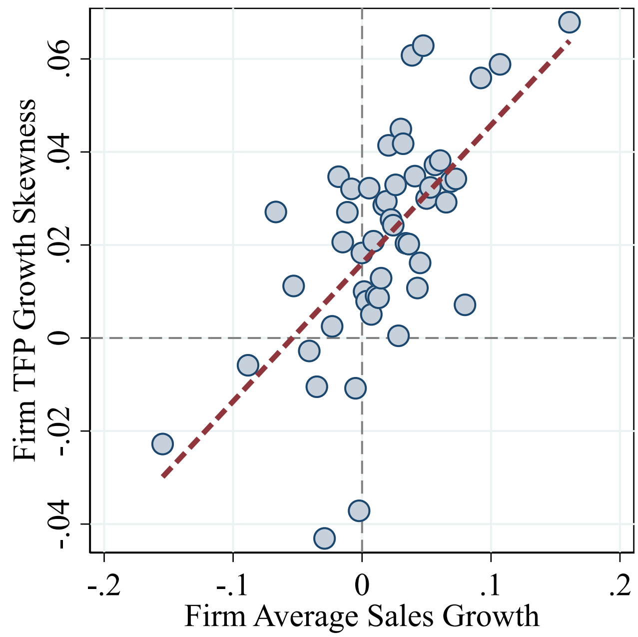

Note: This figure shows the relation between the average firm sales growth within a year-country-industry and the skewness of firm productivity shocks within the same bins calculated using different methods. Binscatters control for year, country, and industry fixed effects. Industries defined as two digit NAICS. The same is constructed using the sample cleaning of GKKV.

Figure B.10 – TFP Measures and Skewness Controlling for Aggregate Conditions

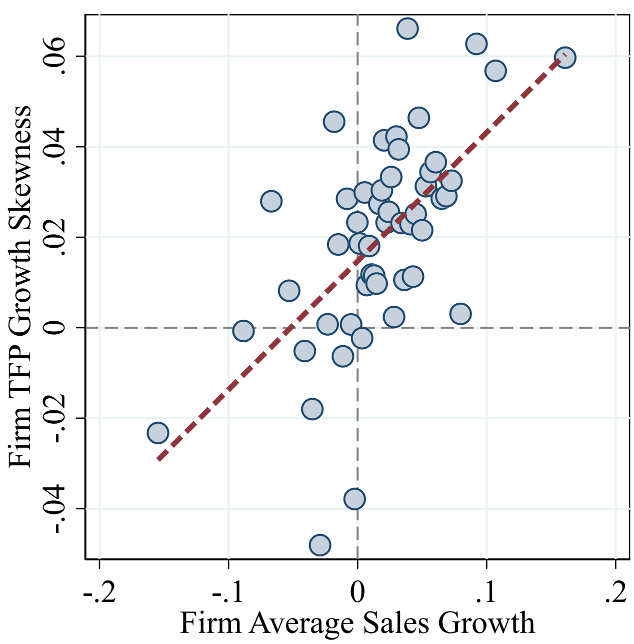
(A) Linear



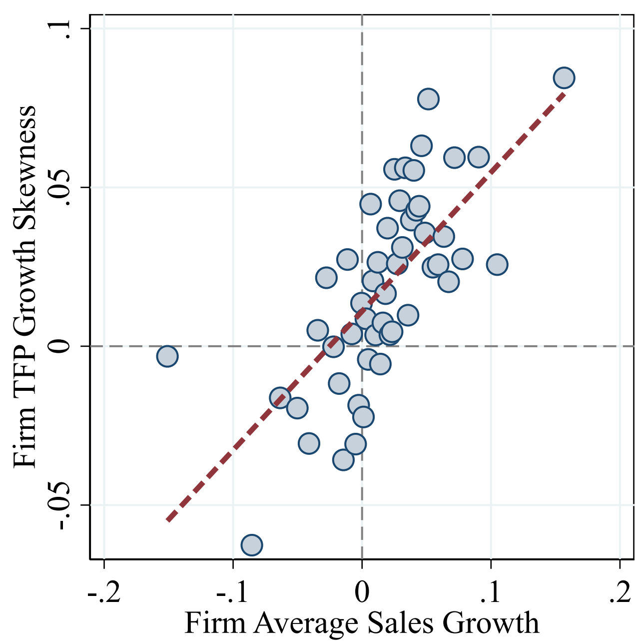

(B) Quadratic




(C) Cubic




(D) Firm-Specific Slopes




Note: This figure shows industry-country-year level binscatters of average sales growth (x-axis) and Kelley skewness of firms TFP innovations (y-axis). Bin scatters control for industry (2-digit NAIC), country, and year fixed effects. Panel A to C are based on Equation 7 for a linear, quadratic, and cubic interaction with country-level GDP growth. Panel D is based on Equation 8 considering firm-specific slopes. TFP estimation is based on the Olley and Pakes (1996) method.

### B.5.6 United States: Productivity, US Census, Survey of Manufacturing, and Capacity Utilization

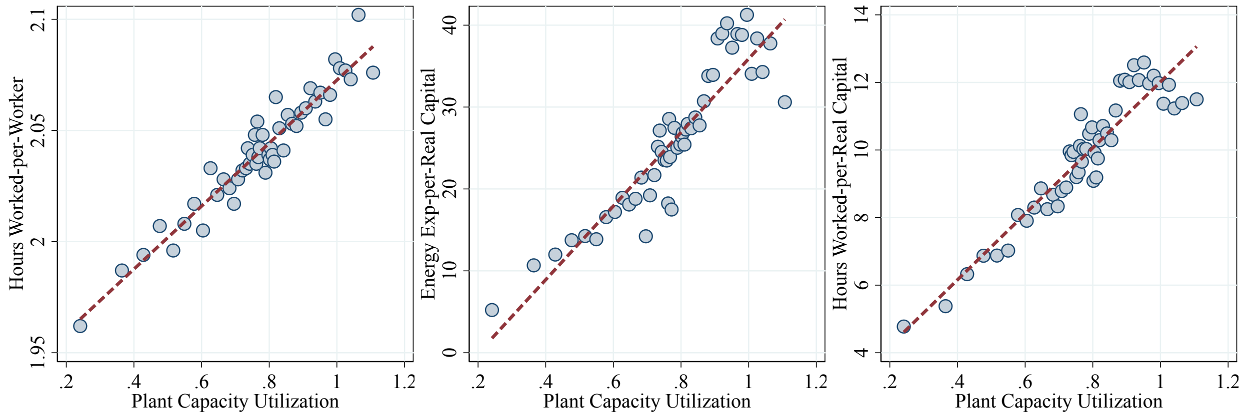
Here we describe the sample selection and moment construction using data from the US Census of Manufacturing (CM) and the Survey of Manufacturing firms (ASM). The CM, which is part of the Economic Census, is conducted every five years, in every year ended in 2 or 5 and was first implemented in 1963. It covers all establishments with at least one paid employee in the manufacturing sector (NAICS 31-33) for a total sample between 300,000 and 400,000 establishments per Census. Information is delivered by firms at the establishment level and Census provides a unique identifier (lbdnum) which we use to follow establishments over time. The Census Bureau complements the CM data with the ASM every year the Economic Census is not conducted since 1973. Relative to the CM, the ASM is skewed towards large firms as it covers all establishments of firms covered by the CM above a certain threshold and a smaller sample of small and medium size firms. The average sample of firms in our sample is around 30,000 establishments per year for a total of about 1.1 million establishment/year observations. The merged CM/ASM contains consistent data in industry, sales, employment, capital expenditures, materials, and others. Importantly, from 1976 to 2019, the data contains measures of log-productivity prepared by the Bureau of Labor Statistics which we directly use in our analysis. The data set and construction of the TFP measures are described in Cunningham, Foster, Grim, Haltiwanger, Pabilonia, Stewart and Wolf (2021).

To keep a consistent sample selection across datasets, we consider establishments for ten or more years of data. Since the ASM sample is refreshed every Census year for small and medium firms, this sample selection criteria naturally select large and stable firms. Our results, however, are robust to the changes in this10-years threshold.

It is well known that failing to control for changes in capacity utilization can lead to wrong estimates on the cyclicality of firm and aggregate productivity (Basu, 1996). This might be even more important using firm-level data and when trying to measure higher order moments. To control for changes in firm-level utilization that might bias our results, we residualize the log of measured productivity for different measures of utilization available in the ASM, namely, average number of hours for productive workers and energy utilization. In particular, we have use firm-level capacity utilization administrative data from the Census' Quarterly Survey of Plan Capacity Utilization (QPC) and the Annual Survey of Manufacturing (ASM). We proceed in three steps which we describe below.

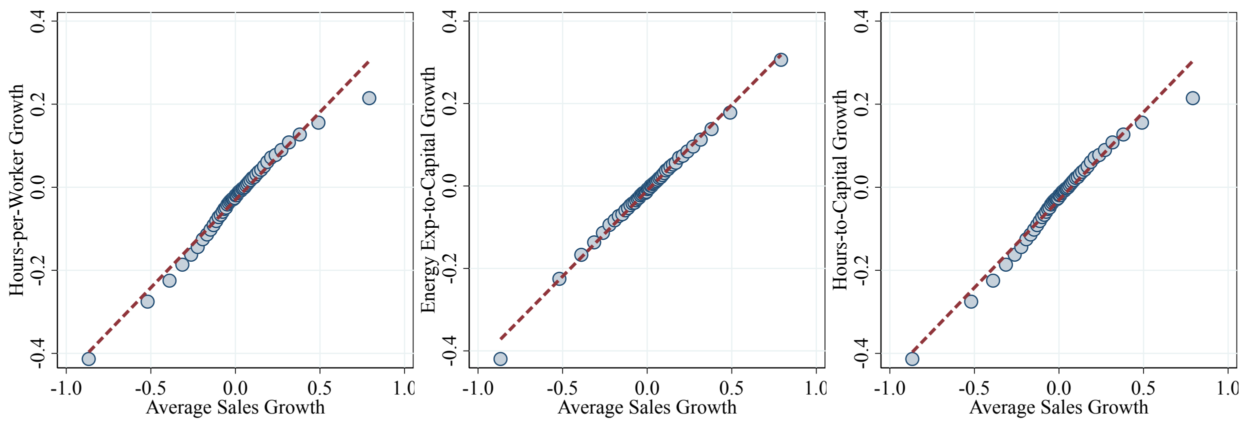
i. First, we use two questions from QPC to calculate what fraction of the total capacity that is used by a plant in any given time. This measure of capacity utilization is only available for those plants in the QPC. Therefore, we merge the QPC data with the ASM which contains information on output and inputs, including energy usage and hours worked. With these variables, we construct three measures of plant capacity utilization: i) average number of hours per worker ii) the ratio between real energy expenditure and real value of machinery and equipment, and iii) the ratio between total hours worked and the real value of machinery and equipment. Figure B.11 shows that all these measures are highly correlated with the QPC measure of capacity utilization, which we take as our benchmark. Hence, in our analysis, we use the ASM-based measures of capacity utilization given they appear to be tightly correlated with QPC data (but which is not available for our fuller

Figure B.11 – Correlation of Capacity Utilization in Census' QPC and ASM




Note: Figure B.11 shows industry-year level bin-scatters the measure of plant capacity utilization from QPC (x-axis) and a proxy for capacity utilization from ASM (y-axis). Binscatters control for year and industry fixed effects. Industry are at the 3-digit NAIC level.

Figure B.12 – Capacity Utilization is Procyclical in ASM




Note: Figure B.12 shows industry-year level bin-scatters the measure of plant capacity utilization growth (y-axis) and plant sales growth (log-change of total value of shipments) from ASM. Binscatters control for year and industry fixed effects. Industry are at the 3-digit NAIC level.

dataset). This delivers three credible measures of capacity utilization for the entire ASM data sample.

ii. Second, we check if the ASM-based measures of capacity utilization are procyclical, which is an important validation step. For this, we use plant-level data and correlate the growth rate of the three ASM-based measures of capacity utilization we described above with the growth rate of the real value of shipments (revenue growth). As expected, we find a positive and strong correlation between revenue growth and capacity utilization growth. These results are displayed in Figure B.12. Each figure shows mean sales growth at the industry level and the mean growth of capacity utilization for different measures controlling for year and industry fixed effects. The tight positive slope indicates that in periods in which the sales growth is above the industry average, utilization is also above its industry average.

iii. Third, using our capacity utilization measures, we regress the log-TFP measure available in Census on these measure of capacity utilization using a rich set of polynomials to account for potential non-linearities in the relation between plant capacity utilization and productivity. This should cleanse any impact of capacity utilization from our measurement of TFP given this flexible functional form. In particular, we use multiple specifications which span a linear term for capacity utilization to a third-degree polynomial. We use the residuals of these regressions as our utilization-adjusted measure of TFP and we analyze whether the skewness of this measure is procyclical. To do so, we calculate the mean and Kelley skewness of TFP growth within a year/3-digit NAIC code and we regress these two measures.

Table B.9 – Skewness of Firm TFP Growth is Procyclical

(a) TFP Growth

| Specification | $\beta$ | SE | *t-stat* | $R^2$ |
|---|---|---|---|---|
| Baseline | 1.158 | 0.061 | 18.75 | 0.131 |
| Residual 1 | 1.164 | 0.062 | 18.75 | 0.129 |
| Residual 2 | 1.164 | 0.062 | 18.74 | 0.129 |
| Residual 3 | 1.167 | 0.062 | 18.78 | 0.129 |
| Residual 4 | 1.168 | 0.062 | 18.8 | 0.129 |
| Residual 5 | 1.164 | 0.062 | 18.91 | 0.133 |
| Residual 6 | 1.151 | 0.062 | 18.68 | 0.130 |
| Residual 7 | 1.168 | 0.062 | 18.8 | 0.129 |

(b) Sales Growth

| Specification | $\beta$ | SE | *t-stat* | $R^2$ |
|---|---|---|---|---|
| Baseline | 0.564 | 0.050 | 11.52 | 0.080 |
| Residual 1 | 0.587 | 0.050 | 11.95 | 0.080 |
| Residual 2 | 0.587 | 0.050 | 11.95 | 0.081 |
| Residual 3 | 0.589 | 0.050 | 11.99 | 0.081 |
| Residual 4 | 0.590 | 0.050 | 12.01 | 0.081 |
| Residual 5 | 0.576 | 0.050 | 11.75 | 0.083 |
| Residual 6 | 0.569 | 0.050 | 11.66 | 0.081 |
| Residual 7 | 0.590 | 0.050 | 12.01 | 0.081 |

Note: Figure B.9 shows industry panel regressions (3-digit NAIC) of the Kelley skewness of TFP growth on the average TFP growth and average sales growth within an industry using data from the Census Annual Survey of Manufacturing. All regressions weighted by sample weights and include a year and industry fixed effects. We use three measures of capacity utilization: Hours per worker, Energy Per Capital, and Hour Per Capital. Baseline is the correlation between TFP shocks skewness and mean TFP shock or mean sales growth without any adjustment for capacity utilization. Residual 1 refers to log-TFP residualized using a model in which the three measures of capacity utilization enter linearly; Residual 2 adds a quadratic term on each measure, Residual 3 adds a cubic term, and Residual 4 adds a quartic term. Residual 5 uses a quartic polynomial only on Hours per worker, Residual 6 a quartic on Energy per Capital, and Residual 7 uses Hours Per Capital unit.

Table B.9a shows the results of these regressions controlling for year and industry fixed effects. The first row (baseline) calculates the relation between TFP shocks skewness and mean TFP shocks or mean sales growth without any controls for capacity utilization. Residual 1 refers to log-TFP residualized using a model in which the three measures of capacity utilization enter linearly; Residual 2 adds a quadratic term on each measure, Residual 3 adds a cubic term, and Residual 4 adds a quartic term. Residual 5 uses a quartic polynomial only on Hours per worker, Residual 6 a quartic on Energy per Capital, and Residual 7 uses Hours Per Capital unit.

We repeat these regressions using the average sales growth within an industry instead of the average TFP growth. The results, shown in Table B.9b, indicate a positive relation between the skewness of TFP growth and the average sales growth at the industry level. We conclude that, although capacity utilization is an important contributor to the observed changes in measure productivity, controlling for its fluctuations does not materially affect the procyclical nature of the skewness of the distribution of firm shocks.

# C Local Projections and Robustness

In this section, we describe in additional detail the data and methods used to estimate the and impulse responses discussed in Section 3.5 calculated using standard local projections (Jordà, 2005) discussed in the main text and we provide some additional robustness results. For our quarterly analysis. We consider a standard set of quarterly macroeconomic variables such as GDP (FRED variable GDPC1), Investment (FRED variable GPDIC1), Consumption (FRED variable PCEC), Employment (FRED variable PAYEMS), and the S&P500 index (closing value of last trading date of the quarter). We also include additional controls such as wages (FRED variable AWHNONAG), CPI, and the Fed Funds Rate. Monthly variables are aggregated to quarterly level by averaging across months with the exception of the Fed Funds Rate (FRED variable FEDFUNDS) for which we take the value at the start of the quarter. All variables are in logs with the exemption of the Fed Funds Rate.

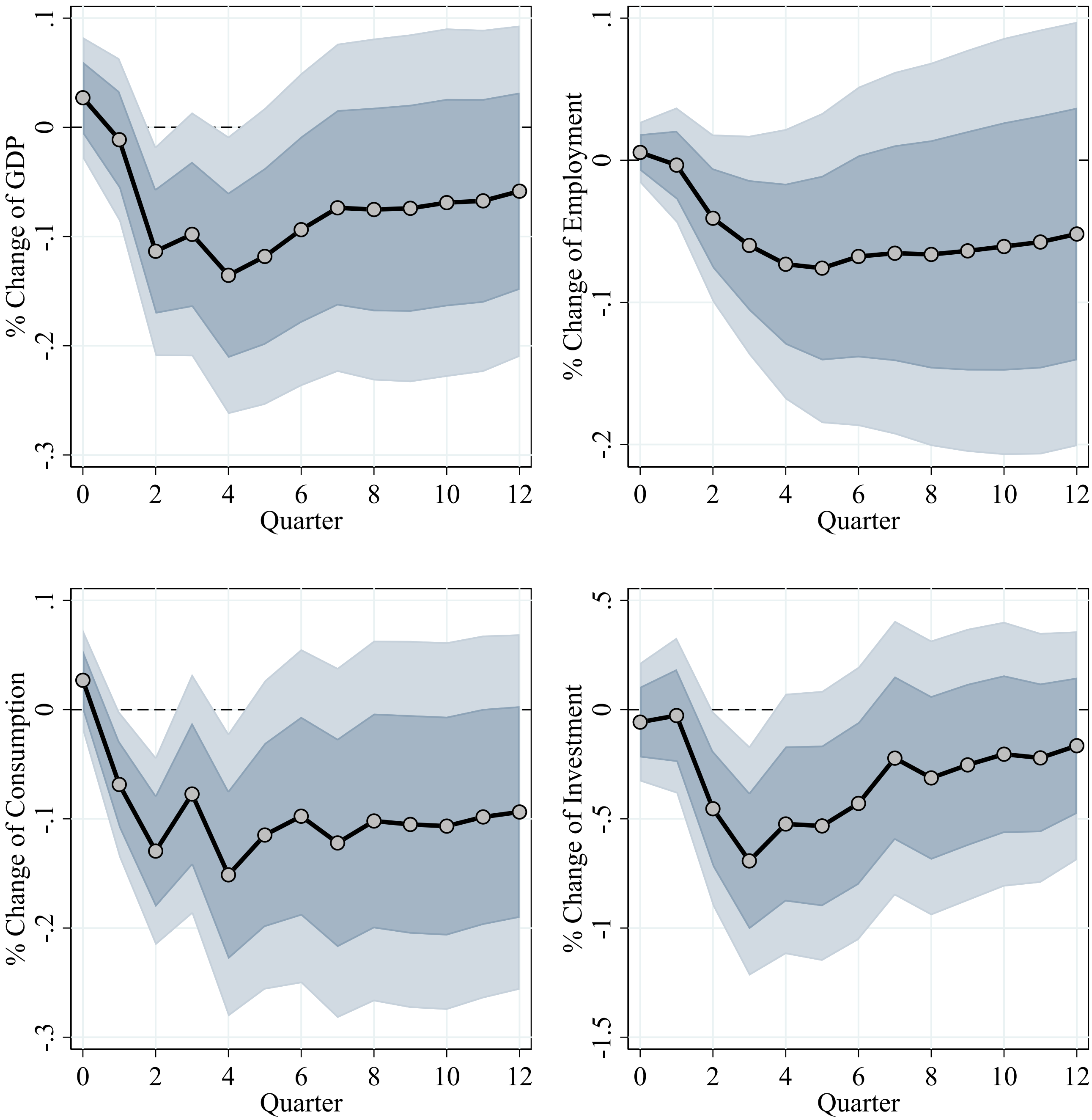
For our monthly-level analysis (presented here in the appendix), we consider the log of the S&P500 stock market index (closing value of last trading date of the month), the Federal Funds Rate (FRED variable FEDFUNDS), the log-level of the average of hourly earnings (FRED variable AHETPI), the log-consumer price index (FRED variable CPIAUCSL), the log-level of hours (FRED variable AWHMAN), the log-level of employment (FRED variable PAYEMS), and the log of an index of industrial production (FRED variable INDPRO).

We construct measures of volatility and skewness using daily returns from publicly traded firms obtained from CRSP dataset acceded through WRDS. In particular, for each firm $i$ we calculate the day-to-day log-change of the stock price within a month $m$, $d_{i,m}$ and then we calculate the difference between the 90th-to-10th percentile differential and the Kelley skewness using all observations of daily returns over all the firms within a quarter or a month, depending on the level of analysis.

To estimate the impact of a skewness shock using local projections, we first residualize the measure of dispersion and skewness of stock returns from contemporaneous or lagged correlation with the same variables use in the analysis and we use these residuals in our analysis. This residualization ensures that our measure the skewness shocks we consider do not capture a mechanical relation of macroeconomic aggregates but an unexpected change in skewness. We consider other measures of skewness, all of which show similar results, as reported in Figure C.14.

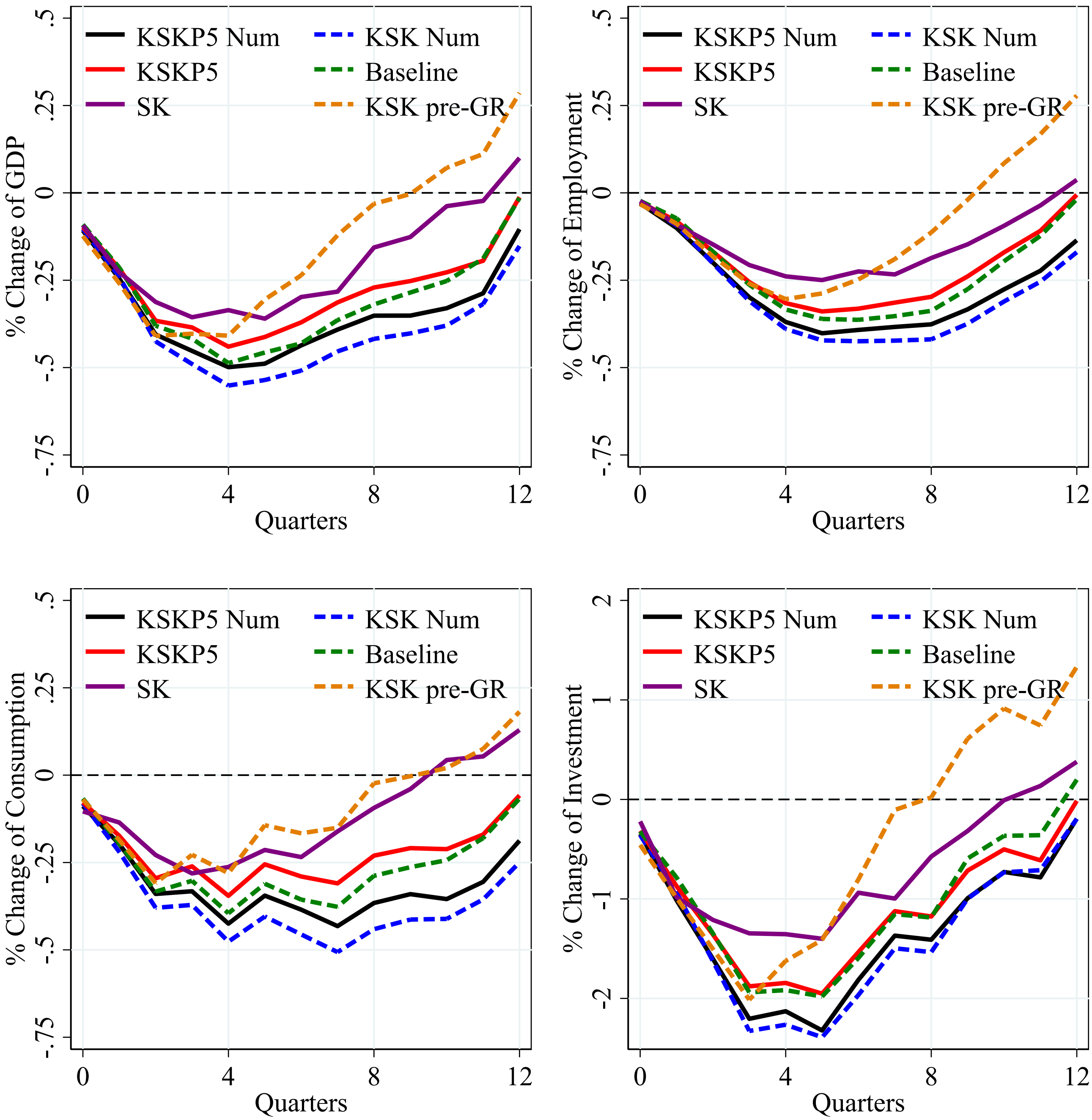
We complement the local projection results using a standard VAR approach where a skewness shock is identified using a Cholesky decomposition. In our baseline results using VAR—presented in Figure C.13—we sorted the measure of skewness third, although the results do not depend on the exact specification used in the analysis. Figure C.15 shows several robustness cases, including changing the order used in the decomposition, excluding the Great Recession, excluding the first and second moments of the distribution of returns, and so on. We repeat our analysis at the monthly level, finding similar results (C.16). I all cases, we have a robust negative impact of a skewness shock.

Figure C.13 – VAR: Macroeconomic Impact of a Skewness Shock




Note: This figure shows the macroeconomic impact of a skewness shocks. The impulse responses were calculated using a standard Cholesky decomposition and show the impact of a one standard deviation decrease in skewness. The light shaded areas represent a 95% confidence intervals.

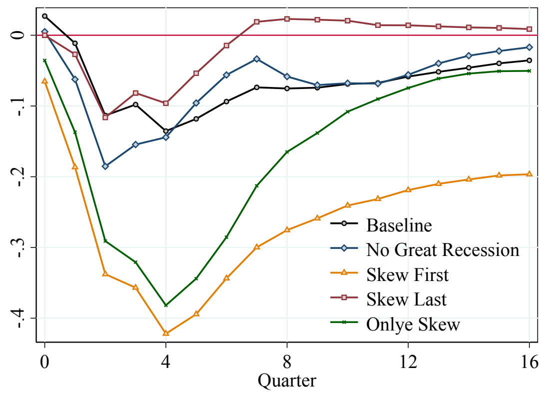
Figure C.14 – Local Projections: Macroeconomic Impact of a Skewness Shock



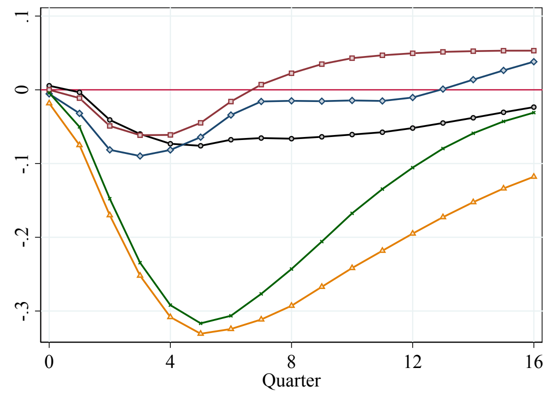

Note: This figure shows the macroeconomic impact of a skewness shocks using different measures of skewness. Skewness shock is the residualized Kelley skewness of daily stock returns. The impulse responses were calculated using a Local Projections approach and show the impact of a one standard deviation decrease in skewness. KSK is the baseline measure of skewness; KSK num uses only the numerator of the Kelley skewness, that is, the difference between the right and left tail dispersion, in this case, calculated as $KSK$ num $= (P90 - P50) - (P50 - P10)$. KSK5 and KSK5 num are the same measures of Kelley skewness but calculated using the $P95$ and $P05$ percentiles. Finally, $SK$ is the coefficient of skewness.

Figure C.15 – Robustness VAR: Macroeconomic Impact of a Skewness Shock

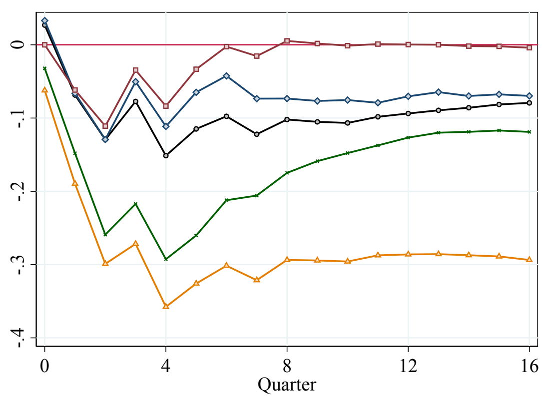
(a) Output



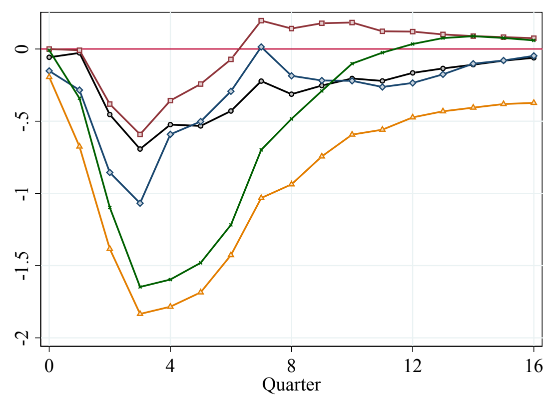

(b) Employment




(c) Consumption



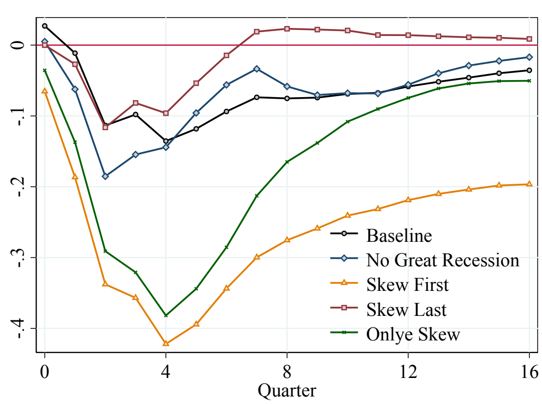

(d) Investment



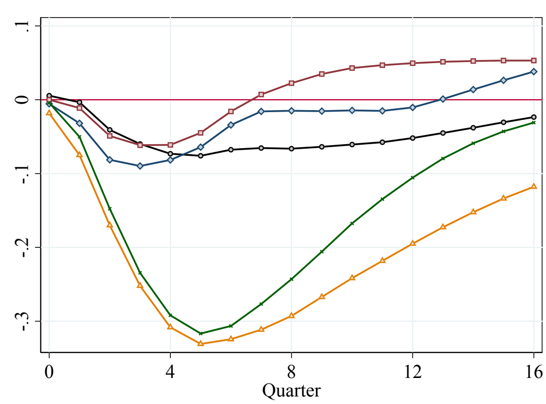

Note: Figure C.15 shows the macroeconomic impact of a skewness shocks. The impulse responses were calculated using a standard Cholesky decomposition and show the impact of a one standard deviation decrease in skewness. The light shaded areas represent a 95% confidence intervals.

Figure C.16 – Robustness VAR: Macroeconomic Impact of a Skewness Shock at Monthly Frequency

(a) Industrial Production




(b) Employment




Note: Figure C.16 shows the macroeconomic impact of a skewness shocks at the monthly frequency.

# D A Quantitative Model

In this appendix we discuss model, calibration, and parameter estimates discussed in Section 4 of the paper.

## D.1 Model Description

**Entrepreneur Households.** Entrepreneurs supply labor to their own firm (they cannot work for someone else's firm), invest in capital, and in a risk-free asset that pays an interest rate $r_t$. Denote the entrepreneur's value function by $V(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$ where $k_{j,t}$ is the entrepreneur's stock of capital, $a_{j,t}$ is the beginning-of-the-period holdings in the risk-free asset, $e_{j,t}$ is the entrepreneur's idiosyncratic productivity, and $\Omega_t$ is a vector of aggregate states.

The entrepreneurs uses capital $k_{j,t}$ and labor $n_{j,t}$ in the production function $y_{j,t} = A_t e_{j,t} k_{j,t}^{\alpha} n_{j,t}^{\nu}$ with $\alpha + \nu < 1$,where $A_t$ is the aggregate productivity that follows a standard the first-order autoregressive process in logs. The idiosyncratic productivity process $e_{j,t}$ is given by

$$e_{j,t} = \rho e_{j,t-1} + \epsilon_{j,t}, \tag{9}$$

where $\epsilon_{j,t}$ has zero mean, time-varying variance, denoted by $\sigma_{\epsilon,t-1}$, and time-varying skewness, denoted by $\gamma_{\epsilon,t-1}$. Consistent with the evidence of firm-level expectations discussed in the main body of the paper (Figure 2), we have assumed that the distribution of innovations in period $t$ depends on the values of the variance and skewness observed in period $t-1$. In other words, the agents know about the distribution of shocks from which the idiosyncratic innovations will be drawn in period $t$ one-period in advance. This timing captures the "news shock" aspect of firm-level risks in the model: an increase in dispersion or a decline in the skewness of firms' shocks represents news about the characteristics of the distribution of innovations in the future but not a concurrent change in the distribution from which the current realizations of $\epsilon_{j,t}$ are drawn. Hence, the vector of aggregates states is $\Omega_t \equiv (A_t, \sigma_{\epsilon,t-1}, \gamma_{\epsilon,t-1}, \mu_t)$ where $\mu_t$ is the distribution of entrepreneurs over idiosyncratic states.

Then, we can write the dynamic problem of the entrepreneur as

$$V(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t) = \max_{\substack{\{c_{j,t}, k_{j,t+1}, \\ a_{j,t+1}, n_{j,t}\}}} \left\{ \frac{c_{j,t}^{1-\xi}}{1-\xi} + \beta \mathbb{E}\left[V(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}; \Omega_{t+1})\right] \right\}, \tag{10}$$

$$\begin{gathered}
\text{s.t. } c_{j,t} + i_{j,t} + a_{j,t+1} \le y_{j,t} - w_t(\Omega_t) n_{j,t} - \phi(k_{j,t+1}, k_{j,t}) + (1 + r_t(\Omega_t)) a_{i,t}, \\
i_{j,t} = k_{j,t+1} - (1-\delta) k_{j,t}, \\
\phi(k_{j,t+1}, k_{j,t}) = \phi_1 \mathbb{I}_{|i_{j,t}|>0} y_{j,t} + \frac{\phi_2}{2} \left( \frac{i_{j,t}}{k_{j,t}} \right)^2 + (1-\phi_3) |i_{j,t}| \mathbb{I}_{i_{j,t}<0}, \\
\mu_{t+1}(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}) = \Gamma(\Omega_t), \\
k_{j,t} > 0, a_{j,t} \ge 0, n_{j,t} > 0,
\end{gathered}$$

given the laws of motion for $A_t$, and the laws of motion of $\sigma_{\epsilon,t}$, and $\gamma_{\epsilon,t}$, which we describe below. The term $w_t \equiv w(\Omega_t)$ denotes the wage rate in the economy. In what follows, we assume the interest rate on the risk-free asset is fixed, that is $r(\Omega_t) = r$.[35] Capital adjustment

[35]This implies that we will not solve the interest rate in equilibrium. The wage rate, however, is such

costs, $\phi(.)$, are given by the sum of a fixed *disruption cost*, $\phi_1$, paid by the entrepreneur for any net investment or disinvestment, a quadratic adjustment cost, $\phi_2$, and a *resale cost* for net disinvestment (partial irreversibility), $\phi_3$. Notice that although entrepreneurs can invest in their own firm or in the risk-free asset, the transformation of resources between assets is not automatic because of adjustment costs. Moreover, entrepreneurs face a borrowing constraint as that they cannot short the risk-free asset to finance the operations of their firms.

**Model Assumptions.** There are a few modeling choices that deserve a further discussion. First, although in our empirical analysis we focus on firm-level data, we assume that the economy is populated by a large number of risk averse entrepreneurs whose income is directly impacted by fluctuations in productivity, and hence, by changes in the skewness of firm shocks. Entrepreneurs have a limited set of asset to insure against idiosyncratic risk which is similar to Cagetti and De Nardi (2006) or Quadrini (2000). In these models, entrepreneurs can self-insure against idiosyncratic productivity shocks. The key assumption, however, is that markets are incomplete, in the sense of standard heterogeneous-agent models a la Bewley-Aiyagari: entrepreneurs face aggregate shocks and idiosyncratic shocks, but are limited to a set of assets (typically one) that do not span the rich set of shocks that can impact them. In our case, entrepreneurs can partially self-insure by either saving in a risky asset—represented by the production technology of each entrepreneur—and a risk-free asset with an un contingent payment—the risk-free asset $a$. Entrepreneurs are subject to a borrowing constraint since $a > 0$ in our exercise, and hence cannot borrow to finance their operations.

This setup might be closer to the problem faced by small firms rather than large corporations that can borrow from the market or issue equity to finance their operations. We preferred to follow this rout for three reasons. First, the vast majority of our data in the United States and across countries pertains to small and medium size firms, whose owners might have difficulties to diversify the idiosyncratic risk associated with running a business. Second, fluctuation in skewness seem to affect in the same magnitude to all firms, independent of their size. In fact, data from Compustat—which only consider large publicly traded firms—or from the Annual Survey of Manufacturing—which is largely populated by large establishments—show similar patters in skewness relative to data from the LBD, which considers all firms in the economy. Hence, we decided to have only one type of (entrepreneurial) firm.

Finally, even though large corporations might be more able to diversify idiosyncratic risk, the managers of these corporations might act as risk averse agents. First, there is ample of evidence that the payoffs of top executives are increasingly linked to the outcomes of the firm they manage, through grants and stock options. For instance, Gabaix and Landier (2008) show that the CEO compensation is linked to the size of the firm. Frydman and Jenter (2010) shows that a large fraction of CEO compensation is options, stocks, bonuses, all of which are connected to the performance of the firm and only 17% of CEO compensation is wages and salaries.[36] Finally, Panousi and Papanikolaou (2012) show that managers and CEOs respond to increases in firm-level idiosyncratic risk by reducing firm investment, and such response increases with the share of ownership that the manager or CEO has in the firm, which is consistent with

that the labor market clears in each period.

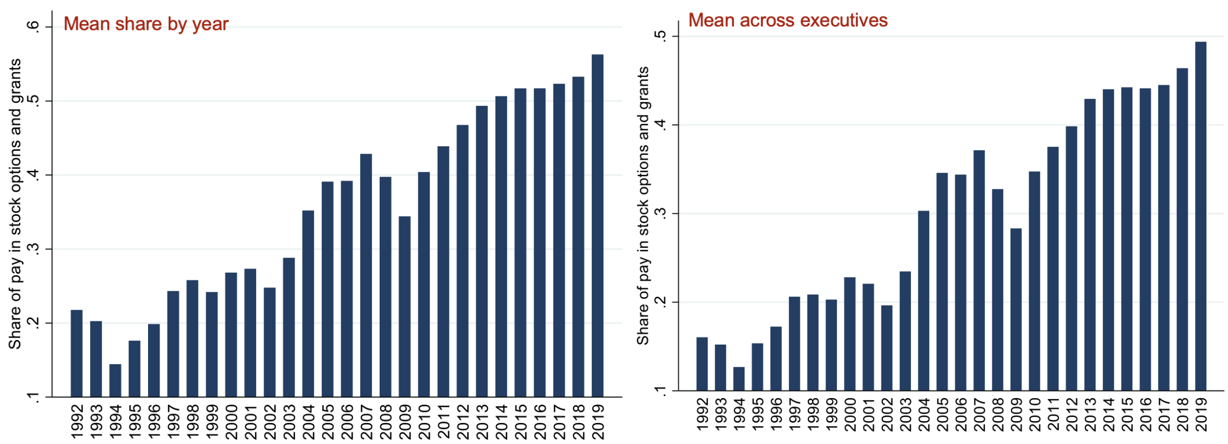
[36]Furthermore, the share of compensation linked to firm outcomes has increased significantly over time, as shown in Appendix Figure D.1 constructed using data from Execucomp. See Edmans and Gabaix (2016) for a review of the theoretical and empirical literature on this subject.

models where managers act as risk-averse agents. In summary, although our model considers entrepreneurs as its main focus on analysis, we think that the insights we learn from it can also be applicable to a broader set of firms, including large corporation.

Figure D.1 – Executives Compensation: Share of Stock Options and Grants

(a) CEO Compensation

(b) Top 5 Executive




Note: This figure shows the average share of CEO compensation coming from stock options and grants for all CEO (left panel) and Top 5 executives (right panel) using data from Execucomp.

**Non-Entrepreneurial Households.** The economy is populated by a large number of identical hand-to-mouth households that consume $C_t$ units of the homogeneous good and supply labor elastically which we denote by $N_t$. This non-entrepreneurial sector chooses consumption and labor to solve the static problem

$$U(C_t, N_t) = \max_{C_t, N_t} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1-\gamma}}{1-\gamma} \right\}, \tag{11}$$
$$C_t \leq w_t N_t,$$

given the law of motion of the aggregate state, $\Omega_t$.

**Recursive Competitive Equilibrium.** Given the exogenous process for aggregate productivity, $A$, the exogenous process of the variance and skewness of $e_j$, the interest rate of the risk-free asset, $r$, and the evolution of the idiosyncratic productivity processes for the entrepreneurs, $\{e_j\}_{j\in J}$, a recursive competitive equilibrium for this economy is a set of policy functions
$\left\{ \left\{ C_j^e, K_j^e, N_j^e, A_{j,}^e \right\}_{j\in J}, C, N \right\}_{t=0}^{\infty}$, a wage function $\{w\}$, and value functions $\{V, U\}$ such that i) the policy and value functions solve (10) and (11), respectively; ii) the labor market clears, that is,

$$\int N^e(k_j, a_j, e_j; \Omega)\, d\mu(k_j, a_j, e_j) = N(\Omega);$$

and iii) the mapping $\Gamma(\omega)$ that determines the evolution of the joint distribution of $e_j$, $k_j$, and $a_j$ is consistent with the policy functions, the evolution of the aggregate productivity process, and the evolution of the process of $\sigma_\epsilon$ and $\gamma_\epsilon$.

## D.2 Parameters and Estimation

Most of the model parameters are standard in the macro literature, and we take them from the existing estimates when possible. The parameters governing the stochastic process of firms' productivity, however, are novel to our analysis, and we use the method of simulated moments to estimate them.

**Frequency, Preferences, and Aggregate Productivity.** We set the time period to be a quarter. For the entrepreneurs, we set a risk aversion parameters, $\xi$, equal to 2.0 and a discount rate, $\beta$, of $0.95^{0.25}$. The interest rate on the risk-free asset is set to match an annual return of 2%. For the non-entrepreneurial sector, we set $\sigma$ to 2. For the labor supply of the non-entrepreneurial households, we fix a value of $\gamma$ to 1.5, and we choose $\psi$ so that they spend an average of 33% of their time working.

The exponents of the capital and labor inputs in the entrepreneur's technology are set to $\alpha = 0.25$ and $\nu = 0.5$. The capital depreciation rate, $\delta$, is set to match an annual depreciation of 14%. As for the adjustment cost parameters, we set the fixed adjustment cost of capital, $\phi_1$, equal to 1.5%, a quadratic adjustment cost, $\phi_2$, equal to 7.0, and a resale cost, $\phi_3$, equal to 34.0%.

We assume that aggregate productivity follows a standard first-order autoregressive process with an autocorrelation of 0.95 and normally distributed innovations with mean 0 and a standard deviation of 0.75%, similar to the quarterly values used in other papers in the literature (Khan and Thomas, 2008).

**Idiosyncratic Productivity.** To capture time-varying risk, we assume that the economy transitions between two aggregate states. The first is a low-risk state (denoted by $L$), which corresponds to periods in which the variance of the innovations of the idiosyncratic shocks is low and the skewness is positive, as we observe during expansion periods. The second is a high-risk state (denoted by $H$), which corresponds to periods in which the variance of the innovations of the idiosyncratic shocks is high and the skewness is negative, as we observe during a typical recession. Low- and high-risk states alternate following a first-order Markov process.

Since high and low risk periods differ in the skewness of entrepreneur's idiosyncratic productivity, we need to depart from the standard assumption of Gaussian shocks. We take a simple approach and assume that, conditional on the risk state of the economy, the innovations of the firms' idiosyncratic productivity process, $\epsilon_{j,t}$, are drawn from a mixture of two normally distributed random variables, that is,

$$\epsilon_{j,t} \sim \begin{cases} N\left(\mu^s, \sigma_1^s\right) & \text{with prob } p^s \\ N\left(-\frac{p^s}{1-p^s}\mu^s, \sigma_2^s\right) & \text{with prob } 1-p^s, \end{cases} \tag{12}$$

where $s$ denotes the risk state of the economy, $s \in \{H, L\}$. Hence, to fully characterize the stochastic process faced by firms, we need to find ten parameters, namely, $\{\mu^s, \sigma_1^s, \sigma_2^s, p^s\}$ with

$s \in \{H, L\}$, and the parameters governing the transition probabilities between low- and high-risk periods, denoted by $\pi_L$ and $\pi_H$, respectively.[37]

Since we do not directly observe the productivity process faced by a large sample of firms in the US economy—our TFP estimates for the United States discussed in Section 3.3 only pertain to a sample of manufacturing firms—we choose the parameters of the stochastic process of firms' productivity to match several features of the distribution of sales growth. In particular, we take data of annual sales growth from LBD, and we search for parameters of the stochastic process so that the cross-sectional distribution of sales growth derived from the model reproduces the observed average values of the 90th-to-50th percentiles differential, the 50th-to-10th percentiles differential, and the 90th-to-10th percentiles differential of the annual sales growth distribution during expansion and recession periods for a total of six moments. The probability of being in the high-risk state in the next period conditional on being in the high-risk state in this period, $\pi_H$, is set to be equal to the fraction of recession quarters that are followed from another recession quarter in the data, $\pi_H = 0.84$, whereas the transition probability of the low risk state, $\pi_L$, is set so that the share of expansion quarters following another expansion quarter is 0.95. Recession and expansion periods in the data correspond to the recession quarters defined by the NBER from 1970 to 2014.

Based on our estimations, we find that in periods of low risk, the variance of the idiosyncratic productivity shocks, is equal to 0.049, whereas the coefficient of skewness is equal to 0.85. In contrast, in periods of high-risk, the variance of the productivity shocks is equal to 0.069, and the coefficient of skewness is equal to -1.14. Table D.1 displays the estimates for the different parameters of the idiosyncratic productivity process, whereas Table D.4 shows the targeted and model-simulated moments.[38] Our model is also consistent with the standard business cycle statistics in terms of the cyclicality and volatility of aggregate output, consumption, investment, and employment (see Table D.2 in the Appendix).

[37]A different approach is to assume that idiosyncratic shocks are drawn from a skewed normal distribution or from a nonlinear transformation of normal shocks as in Orlik and Veldkamp (2014). Alternatively, we could consider a hybrid approach as in McKay (2017) who estimates a labor income process in which innovations are drawn from a mixture of three normally distributed random variables. In his specification, the parameters governing the normal mixture are tied to the aggregate conditions of the economy.

[38]The variance of a random variable $\eta$, which is distributed as a mixture of two normally distributed random variables, is given by $Var(\eta) = \mathbb{E}\left(\eta^2\right) - \mathbb{E}(\eta)^2$, whereas the skewness is given by $Skew(\eta) = \left(\mathbb{E}\left(\eta^3\right) - 3\mathbb{E}(\eta)\, Var(\eta) - \mathbb{E}(\eta)^3\right) / Var(\eta)^{\frac{3}{2}}$. Here $\mathbb{E}(\eta)$ is the first moment of the $\eta$ given by $\mathbb{E}(\eta) = p_1\mu_1 + p_2\mu_2$. Similarly, $\mathbb{E}(\eta)^2 = p_1\left(\mu_1^2 + \sigma_1^2\right) + p_2\left(\mu_2^2 + \sigma_2^2\right)$ and $\mathbb{E}\left(\eta^3\right) = p_1\left(\mu_1^3 + 3\mu_1\sigma_1^2\right) + p_2\left(\mu_2^3 + 3\mu_2\sigma_2^2\right)$ are the second and third moments.

## Table D.1 – Parameterization

| Preferences and Technology | | | Estimated Parameters of Idiosyncratic Stochastic Process | | |
|---|---|---|---|---|---|
| $\gamma$ | 0.45 | Frisch elasticity of labor supply | $\sigma_1^L$ | 1.45 | Standard deviation of first mixture in low-risk periods (%) |
| $\psi$ | 2.5 | Leisure preference, non-entrepreneurs spend 1/3 time working | $\sigma_2^L$ | 7.55 | Standard deviation of second mixture in low -risk periods (%) |
| $\sigma$ | 2.0 | Risk aversion, non-entrepreneurial sector | $\mu^L$ | –0.92 | Mean of first mixture in low-risk periods (%) |
| $\xi$ | 2.0 | Risk aversion, entrepreneurs | $p^L$ | 63.9 | Probability of first mixture in low-risk periods (%) |
| $\beta$ | $0.95^{0.25}$ | Annual discount factor of 95% | $\sigma_1^H$ | 4.37 | Standard deviation of first mixture in high-risk periods (%) |
| $r$ | 0.005 | Annual return of risk-free asset of 2% | $\sigma_2^H$ | 9.06 | Standard deviation of second mixture in high-risk periods (%) |
| $\alpha$ | 0.25 | CRS production, markup of 33% | $\mu^H$ | 1.98 | Mean of first mixture in high-risk periods (%) |
| $\nu$ | 0.50 | CRS labor share of 2/3, capital share of 1/3 | $p^H$ | 78.1 | Probability of first mixture in high-risk periods (%) |
| $\delta$ | 3.8% | Annual depreciation of capital stock of 14.4% | | | Transition Probabilities Across Risk States |
| $\rho_a$ | 0.95 | Quarterly persistence of aggregate productivity | $\pi_L$ | 0.97 | Quarterly probability of remaining in low-risk state |
| $\sigma_a$ | 0.75% | Standard deviation of innovation of aggregate productivity | $\pi_H$ | 0.84 | Quarterly probability of remaining in high-risk state |
| $\rho$ | 0.95 | Quarterly persistence of idiosyncratic productivity | | | |
| | | Adjustment costs | | | |
| $\phi_1$ | 1.5% | Fixed cost of changing capital stock | | | |
| $\phi_2$ | 6.0 | Quadratic cost of changing capital stock | | | |
| $\phi_3$ | 34% | Resale loss of capital | | | |

Note: The top two panels of Table D.1 shows the calibrated parameters referring to preferences, technology, and adjustment costs. The two bottom panels of Table D.1 shows the parameters of the stochastic process of firm-level productivity. We target moments of the annual change of quarterly sales in Compustat. The parameters for low-risk periods (denoted by an upper script $L$) are obtained by targeting the $P9010_t$, $P9050_t$, and the $P5010_t$ percentiles differential, and Kelley skewness of the log sales growth distribution for all the expansion years between 2000 and 2014. The parameters for high-risk periods (denoted by an upper script $H$) are obtained by targeting the same set of moments for years 2001 and 2008 (full recession years). See Table D.4 for comparison between the targeted and model generated moments.

Table D.2 – Business Cycle Statistics

| | Data | | | Model | | |
|---|---|---|---|---|---|---|
| | $\sigma(x)$ | $\sigma(y)/\sigma(x)$ | $\rho(x,y)$ | $\sigma(x)$ | $\sigma(y)/\sigma(x)$ | $\rho(x,y)$ |
| Output | 1.47 | 1.00 | 1.00 | 2.00 | 1.00 | 1.00 |
| Capital Investment | 6.86 | 4.64 | 0.91 | 9.38 | 4.69 | 0.30 |
| Consumption | 1.21 | 0.82 | 0.87 | 1.81 | 0.91 | 0.65 |
| Hours | 1.89 | 1.28 | 0.87 | 2.00 | 1.00 | 1.00 |

Note: The left panel of Table D.2 displays business cycle statistics for quarterly US data covering 1970Q1 to 2017Q4. The column $\sigma(x)$ is the standard deviation of the log variable in the first column. The column $\sigma(y)/\sigma(x)$ is the standard deviation of the variable relative to the standard deviation of log output. All business cycle data are current as of February 3, 2019. Output is real gross domestic product (FRED GDPC1), investment is real gross private domestic investment (FRED GPDIC1), consumption is real personal consumption expenditures (FRED PCECC96), and hours is total non-farm business sector hours (FRED HOANBS). The second panel contains business cycle statistics computed from a simulation of the model of 5,000 quarters with the first 500 periods discarded. All series are HP-filtered with smoothing parameter 1,600, in logs expressed as percentages.

Table D.3 – Targeted Moments for Numerical Comparison

| | $P9010$ | $P9050$ | $P5010$ |
|---|---|---|---|
| Low-Risk | 0.54 | 0.30 | 0.24 |
| High-Risk | 0.70 | 0.31 | 0.39 |
| Only Skewness | 0.54 | 0.243 | 0.297 |

Note: Table D.3 shows the target used in the estimation of the firm-level productivity process. Rows labeled “Low-Risk” and “High-Risk” are used in the baseline estimation. The values for “Only Skewness” are used to estimate the parameters when the economy is shocked with a change in the skewness only.

Table D.4 – Risk Process Moments

| | P90–P10 | P90–P50 | P50–P10 | Years |
|---|---|---|---|---|
| Data | | | | |
| Low-Risk | 0.54 | 0.30 | 0.24 | 03–06;10–14 |
| High-Risk | 0.70 | 0.31 | 0.39 | 01,08 |
| $\Delta\,(H-L)$ | 0.16 | 0.01 | 0.15 | - |
| Model | | | | |
| Low-Risk | 0.48 | 0.27 | 0.20 | - |
| High-Risk | 0.58 | 0.26 | 0.32 | - |
| $\Delta\,(H-L)$ | 0.10 | –0.01 | 0.12 | - |

Note: The top panel of Table D.4 shows cross-sectional moments of the distribution of log quarterly sales growth between quarters $t$ and $t+4$ from Compustat for low-risk periods—quarters in the years 2003 to 2006 and quarters in the years 2010 to 2014—and high-risk periods–quarters in years 2001 and 2008. Quarters in years 2002 and 2009 are discarded for not representing full recession years. The model moments, shown in the lower panel of Table D.4, are calculated from a 5,000-quarters simulation with the first 500 quarters discarded.

## D.3 Solution Algorithm

### Equilibrium Mapping and Algorithm

Given these choices, the evolution of the aggregate equilibrium can be fully characterized by the mappings,

$$\begin{aligned} w_t\left(\Omega_t\right) &= \Gamma_w\left(\Omega_t\right)=\Gamma_w\left(A_t,\sigma_{\epsilon,t-1},\gamma_{\epsilon,t-1},\mu_t\right), \\ \mu_{t+1}\left(\Omega_t\right) &= \Gamma_\mu\left(\Omega_t\right)=\Gamma_\mu\left(A_t,\sigma_{\epsilon,t-1},\gamma_{\epsilon,t-1},\mu_t\right). \end{aligned}$$

There are four main challenges when solving the problem in (10) and the equilibrium mappings, $\Gamma_w$ and $\Gamma_\mu$. The first is the large idiosyncratic state space, which consists in the idiosyncratic productivity shock, $e_{j,t}$, the holdings on capital, $k_{j,t}$, and the holdings on the risk-free asset, $a_{j,t}$. Second, the cross-sectional distribution of entrepreneurs over idiosyncratic states, $\mu_t$, is usually a large and intractable state variable. Third, the number of aggregate state variables is quite large, since not only the aggregate productivity but also the variance and skewness of distribution of idiosyncratic productivity shocks are part of the aggregate state space. Fourth, the equilibrium mapping for wages $\Gamma_w$ must be also approximated and solved to be consistent with the clearing of the labor market.

We address each of these issues as follows. Given an aggregate state of the economy and levels for $a_{j,t-1}$, $k_{t,j-1}$, and $e_{j,t}$, the labor demand of the entrepreneur is fully flexible and can be easily characterized by solving a simple first-order condition. However, the solutions for $k_{j,t}$ and $a_{j,t}$ are more complicated and time consuming, especially if one solves the problem allowing the entrepreneur to choose continuously over the state space. To render the problem more tractable, we solve the problem of the entrepreneur over a grid of points for $k_{j,t}$ and $a_{j,t}$.

We increase the number of points on the grid until our results do not change further increasing the number of points.

As for the variance and the skewness of the idiosyncratic productivity shocks, we assume that a single two-state Markov process $s \in \{H, L\}$ for risk governs the evolution of the second and third moment of $e_{j,t}$ across two possible risk levels ($e_{j,t}$ is mean-zero): if the economy is in the $H_t$ state, or high risk state, the variance of the shocks is high and the skewness is negative; instead, if the economy is in the $L_t$ state, or low risk state, the variance of the shocks is low and the skewness is positive. As we described in more details in Section D.6, conditional on the state, we assume that the innovations of the stochastic process for $e_{j,t}$ are drawn from a mixture of two normally distributed random variables. Hence, the pair $(\sigma_{\epsilon,t}, \gamma_{\epsilon,t})$ can take two values $(\sigma_{\epsilon,t}, \gamma_{\epsilon,t}) = (\sigma_{\epsilon,H}, \gamma_{\epsilon,H})$ or $(\sigma_{\epsilon,t}, \gamma_{\epsilon,t}) = (\sigma_{\epsilon,L}, \gamma_{\epsilon,L})$ with transition matrix given by

$$\Pi^S = \begin{bmatrix} \pi_L & 1-\pi_L \\ 1-\pi_H & \pi_H \end{bmatrix},$$

where $\pi_L$ is probability of stay in the low risk state conditional being in the low risk-state whereas $\pi_H$ is the conditional probability of staying in the high risk state.

We then follow the bulk of the literature and we approximate the cross-sectional distribution, $\mu_t$, with the end-of-the-period aggregate capital level, given by $K_{t+1} = \int k_t\left(k_{j,t-1}, a_{j,t-1}, e_{j,t}; \Omega_t\right) d\mu_t$, the level $A_t$, the square of $A_t$, and the lagged risk state, $s_{t-1}$. Given these changes, the approximated aggregate state vector is given by $\Omega_t \equiv (A_t, s_{t-1}, K_t)$. This allows us to eliminate the distribution of idiosyncratic state and one of the aggregate state variables.

We now can define an approximation to the equilibrium mappings $(\Gamma_w, \Gamma_\mu)$ which we replace by the log-linear rules $\left(\hat{\Gamma}_w, \hat{\Gamma}_K\right)$ :

$$\begin{aligned} \hat{\Gamma}_w: \quad \log w_t &= \alpha_{w,1}(s_{t-1}) + \alpha_{w,2}(s_{t-1}) \log A_t + \alpha_{w,2}(s_{t-1}) \log A_t^2 + \alpha_{w,2}(s_{t-1}) \log K_t \\ \hat{\Gamma}_K: \quad \log K_t &= \alpha_{K,1}(s_{t-1}) + \alpha_{K,2}(s_{t-1}) \log A_t + \alpha_{K,2}(s_{t-1}) \log A_t^2 + \alpha_{K,2}(s_{t-1}) \log K_t, \end{aligned} \tag{13}$$

where the dependence of each parameter on $S_{t-1}$ indicates that we calculate one set of parameters for each risk state of the economy.

The conditions for $\hat{\Gamma}_w$ and $\hat{\Gamma}_K$ give us an approximated equilibrium, which we can then use to lay out the solution algorithm of our model. We start by assuming an approximate mapping $\hat{\Gamma}_w^{(1)}$ and $\hat{\Gamma}_K^{(1)}$ and we guess a set of coefficients for the system in expression (13). Then, we perform the following steps in each iteration $q$:

- *Step 1: Solving the problem of the entrepreneurs*
  Solve the problem of the entrepreneurs in (10) after replacing the approximate equilibrium conditions $\hat{\Gamma}_w^{(q)}$ and $\hat{\Gamma}_K^{(q)}$ using Value Function Iteration; This results in a value function of the entrepreneur, which we denote by $\hat{V}^{(q)}$.

- *Step 2: Simulating the model*
  Using the approximated value function of the entrepreneur, simulate a panel of $N$ entrepreneurs for $T$ periods without imposing the forecasting rules. Importantly, in each period we solve for the wage level that clears the labor market.

- *Step 3: Update the approximate mapping*
  Use the simulated data to construct the log of wages and the log of aggregate capital and estimate the $\alpha_w$ and $\alpha_K$ parameter running a OLS regression conditional on the risk state of the economy, $S_t = \{H, L\}$, denote the estimated forecasting rules by $\tilde{\Gamma}_w^{(q)}$ and $\tilde{\Gamma}_K^{(q)}$.

- *Step 4: Testing convergence*
  If $\tilde{\Gamma}_w^{(q)}$ and $\tilde{\Gamma}_K^{(q)}$ are close enough to $\hat{\Gamma}_w^{(q)}$ and $\hat{\Gamma}_K^{(q)}$, i.e., the maximum absolute difference is below a predefined level of tolerance, exit the algorithm; Otherwise, go to Step 1 using $\hat{\Gamma}_w^{(q+1)} = \theta_\beta \hat{\Gamma}_w^{(q)} + (1-\theta_\beta)\,\tilde{\Gamma}_w^{(q)}$ and $\hat{\Gamma}_K^{(q+1)} = \theta_\beta \hat{\Gamma}_K^{(q)} + (1-\theta_\beta)\,\tilde{\Gamma}_K^{(q)}$ as new guesses and run a new iteration, $q+1$, with a value of $\theta_\beta = 0.75$.

This general algorithm allows us to characterize the problem of the entrepreneur and the equilibrium solution of the model. Each step, however, requires several numerical choices that we now discuss in further detail.

### The Problem of the Entrepreneur

We solve the problem of the entrepreneur over a discrete grid of points. For the capital grid, $k_{j,t}$, we choose a log-linear grid with $n_k = 123$ points closed with respect to the capital depreciation rate. This ensures that firms can always adjust their capital at no cost if they set investment equal to 0. As for the risk-free asset, $a_{j,t}$, we choose a linear grid of $n_a = 43$ points. We discretize the exogenous productivity process, $A_t$, following the standard method of Tauchen (1986) using $n_A = 5$ points. We also discretize the idiosyncratic productivity process, $e_{j,t}$, using a modified version of the method of Tauchen (1986) that allows for a mixture of normally distributed random variables over a grid of $n_e = 11$ points centered around 0. We provide more details on this discretization in Section D.6. As for the grid of aggregate capital, $K_t$, we choose an equally spaced grid of $n_K = 15$ after ensuring that adding additional points doe not alter our results significantly.

Given the discretization of the problem of the entrepreneur, we solve for the fixed point of $\hat{V}^{(q)}$ using Value Function Iteration and a Howard policy iteration of 50 steps (see Judd (1998)). Continuation values are computed using linear interpolation in the direction of the aggregate capital, $K_t$, over the value of $K_{t+1}$ implied by the mapping implied by $\hat{\Gamma}_K^{(q)}$. Although the method allows for the exact calculation of the policy functions—which in general converge quite fast—the period-by-period solution of the equilibrium requires an accurate approximation of the continuation value of the entrepreneurs.

### Montecarlo Simulation and Equilibrium Solution

We simulate the model using a fixed set of $N = 2000$ entrepreneurs for $T = 5000$ periods for which we have drawn aggregate productivity levels, risk realizations, and idiosyncratic productivity shocks, following the discrete Markov approximations discussed above. In practice, using a panel of entrepreneurs to track the distribution $\mu_t$ is time consuming and generates stochastic sampling error that can affect our results. To address these issues we increase the number of individuals in our simulation until our results do not change substantially.[39]

[39] There are several different alternatives to keep track of the distribution of entrepreneurs over idiosyncratic states. For instance, one could use an histogram method as in Young (2010). Although

In each period of the simulation step we make sure the policy functions on capital and labor are consistent with market clearing as well as entrepreneur's optimization. That means that in every period the demand for labor coming from the entrepreneurs must be equal to the supply of labor generated by the non-entrepreneurial household. To make sure this is the case, in each period we disregard the wage forecasting rule $\hat{\Gamma}_w^{(q)}$ and, $\tilde{w}_t$, we iterate over a market clearing wage. In particular, for any guess of the wage rate we solve the solve for each entrepreneur the right-hand-side of the problem in 10 replacing $w_t(\Omega_t)$ by $\tilde{w}_t$ and the continuation value by $\hat{V}^{(q)}$ interpolated over the next period's aggregate capital generated from $\hat{\Gamma}_K^{(q)}$. The solution of this problem gives us a labor demand for each entrepreneur, $\tilde{N}_{j,t}^{e(q)}$. Hence, market clearing is reached when aggregate labor demand $\int \tilde{N}_{j,t}^{e(q)} \mu_t$ is equal to the supply of labor derived from the solution of the non-entrepreneurial household. That is, for a given level of wage, the aggregate demand for labor must be equal to $N_t^q = \left(\psi/\tilde{w}_t^{1-\sigma}\right)^{\gamma-\sigma}$. In practice, to obtain market clearing, we use a simple bisection approach and a error tolerance of $10^{-4}$.

### Update of the Equilibrium Mapping

At the end of the $T$- periods simulation for iteration $q$ we have obtained a time series of wages, aggregate capital stock, and a panel of firm-level outcomes given a guessed mapping $\left(\tilde{\Gamma}_w^{(q)}, \tilde{\Gamma}_K^{(q)}\right)$. To update the equilibrium mapping we discard the first 500 periods and we separate the time series conditional on their risk-state, $S_t$. We then obtain the updated mapping $\left(\tilde{\Gamma}_w^{(q)}, \tilde{\Gamma}_K^{(q)}\right)$, by simply running a set of OLS regressions over the simulated data. Then we compare $\left(\tilde{\Gamma}_w^{(q)}, \tilde{\Gamma}_K^{(q)}\right)$ to $\left(\hat{\Gamma}_w^{(q)}, \hat{\Gamma}_K^{(q)}\right)$. In the case the maximum absolute difference is about certain predefined level, we set $\left(\hat{\Gamma}_w^{(q+1)}, \hat{\Gamma}_K^{(q+1)}\right) = \left(\tilde{\Gamma}_w^{(q)}, \tilde{\Gamma}_K^{(q)}\right)$ and restart the algorithm with a new guess of the equilibrium mapping.

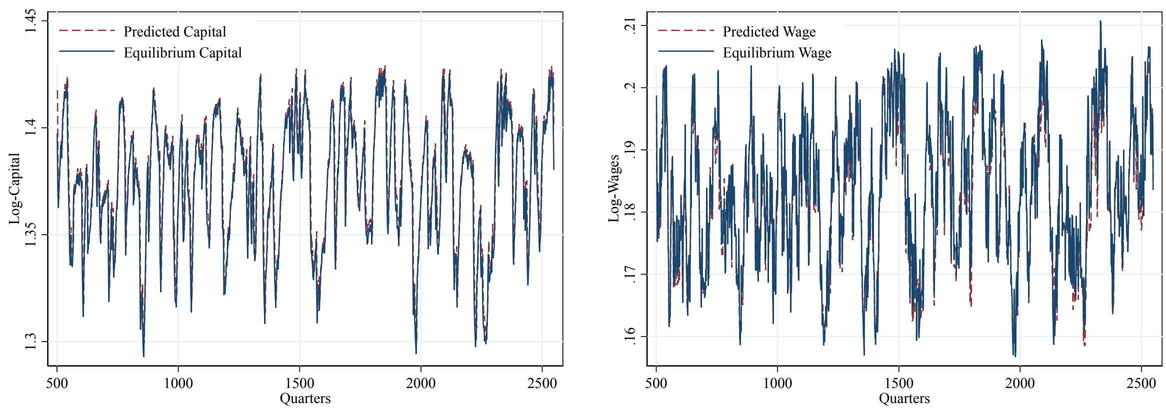
## D.4 Accuracy Tests

The algorithm described in the previous section only provides an approximation of the true path of equilibrium prices and forecasting rules. Hence, it is necessary to test whether the approximate mapping used to solve the problem of the entrepreneurs serves as an accurate forecasting rule of the aggregate capital and wage. There is no a unique way to measure the accuracy of the forecasting rules. Hence, here we discuss two standard accuracy tests. First, we have that the $R^2$ of the regression used for updating the equilibrium mapping is above 97% and the root mean square error (RMSE) of the regressions is below 0.1% for all specifications. As noted by Den Haan (2010), however, the accuracy test based on static metrics like the $R^2$ or the RMSE are not good to measure the accuracy of the forecasting rules. Instead, he proposes using dynamic forecasts that compares the model simulated time series for wages and capital, $(w_t, K_t)$, to their counterparts forecasted using the approximate mapping $\left(\hat{\Gamma}_w^{(q)}, \hat{\Gamma}_K^{(q)}\right)$ $s$-periods ahead. Figure D.2 shows the equilibrium level and forecasted value for capital and wages for a typical simulation of our model. As we can see, the evolution of both aggregates is tracked very well by the approximate mapping $\left(\hat{\Gamma}_w^{(q)}, \hat{\Gamma}_K^{(q)}\right)$ with a average absolute difference between the forecasted and true equilibrium level of 0.6% (standard deviation of 0.7%) for capital; for

faster, the resulting histogram does not allow the fast computation of the distribution of the growth rate of sales or employment, both of which are necessary for our analysis.

Figure D.2 – Evolution of Predicted and Equilibrium Aggregates

(a) Aggregate Capital (b) Wages




Note: The left panel of Figure D.2 shows the evolution of the aggregate capital generated by the model and the predicted capital generated by the approximated mapping, $\hat{\Gamma}_K$. The left panel shows similar results for the equilibrium wage.

the equilibrium wage the average absolute difference is 0.1% (standard deviation of 0.1%).

## D.5 Impulse Response

In this section we provide additional details on the calculation of the response of our model to a change in risk and the rest of the experiments we present in the main body of the paper.

To compute the conditional response of a change in risk we take the resulting forecasting rules from the algorithm discussed in Section D.3 and we simulate 1,000 independent economies of 300 periods each. This lengths ensures that the distribution imposed to initialize the simulation does not influence our results. In each economy $i$, we assume that the aggregate shock $A_t = 1$ and stay constant for the entire simulation. Furthermore, we assume that economy is in the low risk state (low volatility and positive skewness) between periods 1 and $T_{shock} - 1$. Then, in period $T_{shock} = 150$ we impose the high risk state (high volatility and negative skewness); thereafter, each economy evolves normally for the remaining periods.

At the end of the simulation we obtain a panel of aggregate time series, one per each simulated economy. We then average the value of each macro aggregate (e.g., output, investment, dispersion of sales growth, skewness of employment growth, etc.) across all simulated economies and we calculate, for macroeconomic aggregates, the response of variable $\overline{X}_t$ to a change in skewness in period $T_{shock}$ as $\hat{X}_t = 100 \times \log\left(\frac{\overline{X}_t}{\overline{X}_{T_{shock-1}}}\right)$. As for the cross-sectional moments of the sales growth and employment growth distributions—which are normally expressed in percentage points and/or can take negative values—we simply calculate $\hat{X}_t = \left(\overline{X}_t - \overline{X}_{T_{shock-1}}\right)$.

## D.6 Normal Mixture

We conclude with a discussion of the method we use to approximate the stochastic productivity process of the entrepreneurs. Our main empirical results suggest that the productivity shocks affecting firms have time varying skewness, which become negative during recessions. Civale *et al.* (2015) have show, however, that a standard AR(1) process with normally distributed innovations does not do a good job in accounting for the cyclicality of the skewness of wage growth observed in the data (Guvenen *et al.*, 2014). Given these considerations, in order to account for the negative (positive) skewness of productivity shocks observed during recession (expansion) years, we assume that the productivity innovations are drawn from a mixture of two normally distributed random variables. In particular, we assume that in the process of $e_{i,t}$ given by

$$e_{j,t} \quad = \quad \rho_e e_{j,t-1} + \eta_{j,t},$$

the level of $\eta_{j,t}$ is drawn from

$$\epsilon_{j,t} \sim \begin{cases} N\left(\mu^s, \sigma_1^s\right) & \text{with prob } p^s \\ N\left(-\frac{p^s}{1-p^s}\mu^s, \sigma_2^s\right) & \text{with prob } 1-p^s, \end{cases} \tag{14}$$

where $s \in \{H_t, L_t\}$. Hence, for a given level of the aggregate risk, we need to determine four parameters, $\{\mu^s, \sigma_1^s, \sigma_1^s, p^s\}$. Notice we have not assumed that $e_{j,t}$ is log normal, but normally distributed instead. This assumption is useful as it ensures that the mean of the productivity process does not change with variations in the volatility or the skewness of $\eta_{j,t}$. If we were to assume, instead, that the innovations are log-normally distributed, changes in the variance of $\eta_{j,t}$ will impact the mean of $e_{j,t}$ confounding the effects of a first and second moment shocks. The main drawback, however, is that $e_{j,t}$ can now take negative values. In practice, our modification of the method of Tauchen (1986) ensures the grid of $e_{j,t}$ that we use to solve the problem of the entrepreneurs is always positive. In the simulation, however, we assume that $e_{i,t}$ follows a continuous process—and we interpolate the value function using linear interpolation—but we impose that the productivity always takes values within the boundaries of the same grid we used to solve the problem of the entrepreneurs by replacing value below the minimum (maximum) point of the grid by the minimum (maximum) value of the grid. Given our grid is fairly wide, these events are very rare and occur for less than 0.01% of the total number of firm/period observations used in the simulation.

## D.7 Idiosyncratic Shocks and Model Fit

To evaluate the effects of a decrease in the skewness of firm-level shocks, we independently simulate 1,000 economies, each of 300 quarters' length. For the first half of the simulation, all the simulated economics are in the low-risk state, and then in period $T$, all economies are hit by a change in the level of risk. From that period on, we let all economies and stochastic processes to evolve normally. We then average different macroeconomic outcomes across all simulated economies and calculate the impact of the change in risk as the log percentage deviation of a given macro variable relative to its value in the period previous to the shock.

We begin by analyzing the response of the distribution of firm productivity growth after a change in aggregate risk. The left panels of Figure D.3 display moments of the distribution of firms' idiosyncratic productivity growth, $\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$, for three different cases cases. In the first case, the economy moves from the low-risk state to the high-risk state, leading to an increase in the variance and a decrease in the skewness of idiosyncratic shocks (blue line with circles), which corresponds to what it is observed during a typical recession. In the second case, the increase in risk leads to a decrease in the skewness of idiosyncratic shocks only (black line with diamonds), and finally, in the third case, the increase in risk leads to an increase in the variance of idiosyncratic shocks only, which is the typical uncertainty shock studied in the literature (red line with triangles). The top left panel of Figure D.3 shows that the average firm in our model does not experience a change in productivity when risk changes. This ensures that our results are not driven by a change in average productivity and are driven solely by changes in the shape of the distribution of productivity shocks. Then, comparing the black line in the middle and bottom left panels, one can see that our model is able to generate a pure change in the skewness, that is, a change in the productivity distribution that reflects only a decrease in the skewness but a muted change in the mean and the variance of the firm-level productivity distribution.[40] Similarly, our model can generate a pure uncertainty shock (the red line with triangles in the middle panels of Figure D.3).

We now analyze the response of the sales growth distribution—our empirical target—to a change the variance and skewness of firms' shocks. The right panels of Figure D.3 show the average, the dispersion, and the skewness of the annual change in quarterly sales implied by the model calculated as $\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$. It is not surprising that a change in risk that combines a simultaneous increase in the variance and a decrease in the skewness of firm-level productivity shocks generates an increase in the cross-sectional dispersion of sales growth and a large decrease in skewness (blue line with circles in the middle and bottom right panels). Comparing the case in which only dispersion changes—which is the typical uncertainty shock—with the case in which only the skewness changes—the baseline case we discuss in the following section—one can see that by considering a shock with time-varying skewness, the model is able to capture the asymmetric response of the tails of the sales growth distribution (compare the red line with triangles to the blue line with circles in the bottom right panel).

Figure D.4b shows that the Kelley skewness of the employment growth distribution also declines after the drop in the skewness of firms' shocks. Also importantly, Figure D.4a shows that the dispersion and the skewness of sales growth do not change after a decline in aggregate productivity, $A_t$, indicating that aggregates changes in productivity are not likely to drive drop in the skewness of outcomes in our model.

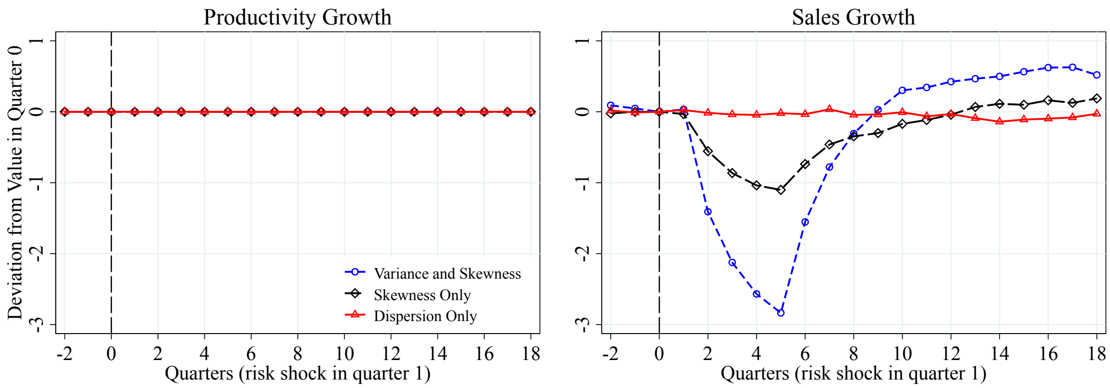
## D.8 Robustness

We perform a series of robustness checks to evaluate how the impact of a skewness shock changes under different parameterizations. In all cases, we keep the parameters of the stochastic process fixed, but we re estimate the firm decision and forecasting rules. The results are shown in Figure D.5. Although there is quite a bit of variation across different cases, in all of them, the skewness shock generates a decline in aggregate economic activity which is significant and persistent (top panel) and this effect is re enforces when combined with a variance shock (bottom panel).

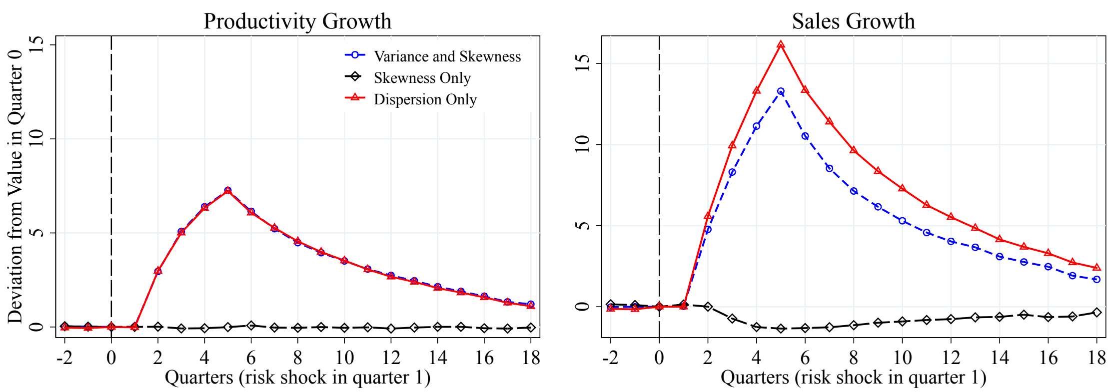
[40]The median firm, however, experiences an increase in productivity after a decline in the skewness. Disentangling the mean and the median of the distribution of firms' shocks allow us to keep the mean and variance constant after a change in skewness.

FIGURE D.3 – PRODUCTIVITY AND SALES GROWTH AFTER AN INCREASE IN RISK

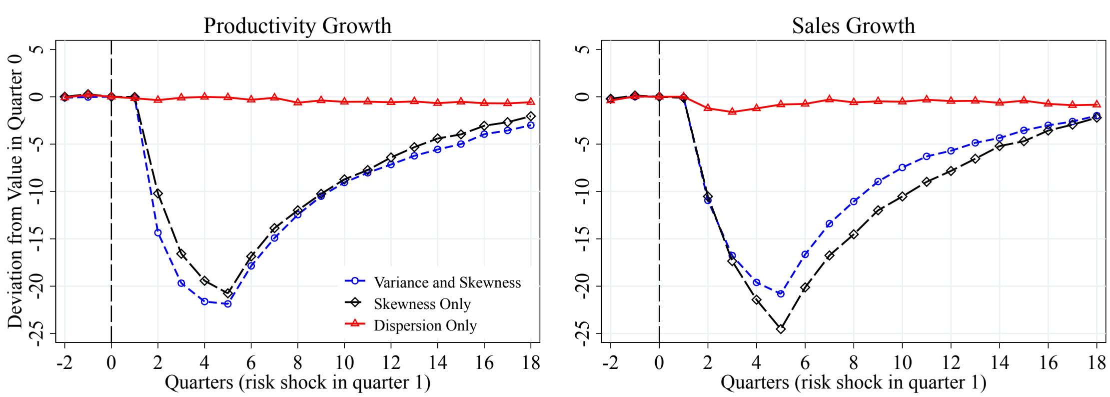
(A) Average




(B) P90-P10



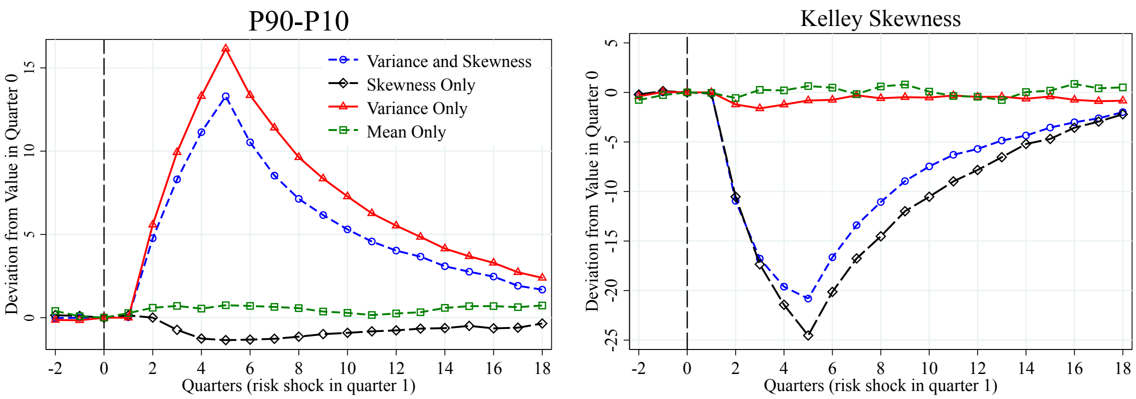

(C) Kelley skewness



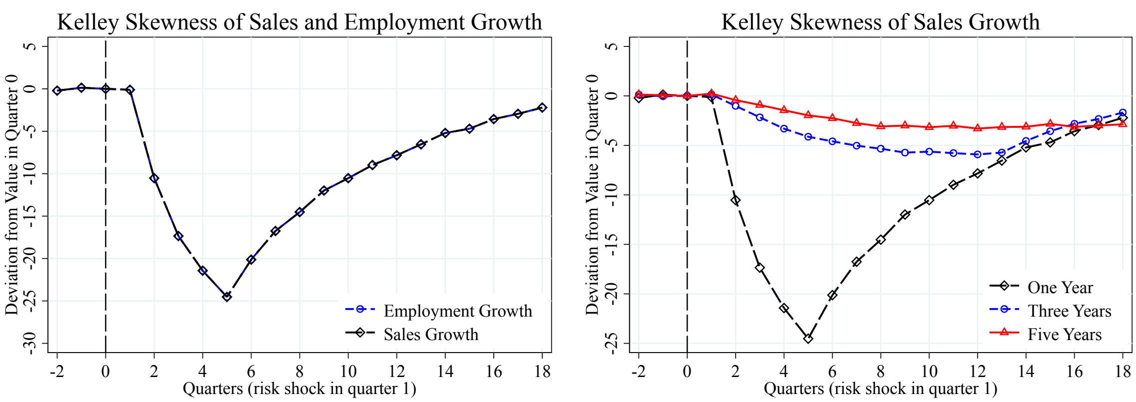

Note: The top left panel of Figure D.3 shows the model-generated average of the one-year productivity growth distribution ($\Delta e_{j,t} = e_{j,t+4} - e_{j,t}$), whereas the top right shows the average of the log sales growth distribution ($\Delta y_{j,t+4} = \log y_{j,t+4} - \log y_{j,t}$) for different risk shocks. The middle and bottom panels show the dispersion and skewness. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. In each simulation, we assume that the economy is in the low-risk state for 150 periods. We then impose a risk shock in quarter 151, allowing normal evolution of the economy afterwards. We plot the deviation relative to the moment value in quarter 0. The red line with triangles trances the impact of an increase in the variance of firms' shocks; the black line with diamonds trances the impact of a drop in the skewness of firms' shocks; the blue line with circles trances the joint impact of an increase in variance and a decrease in skewness.

## Figure D.4 – Model-Generated Moments

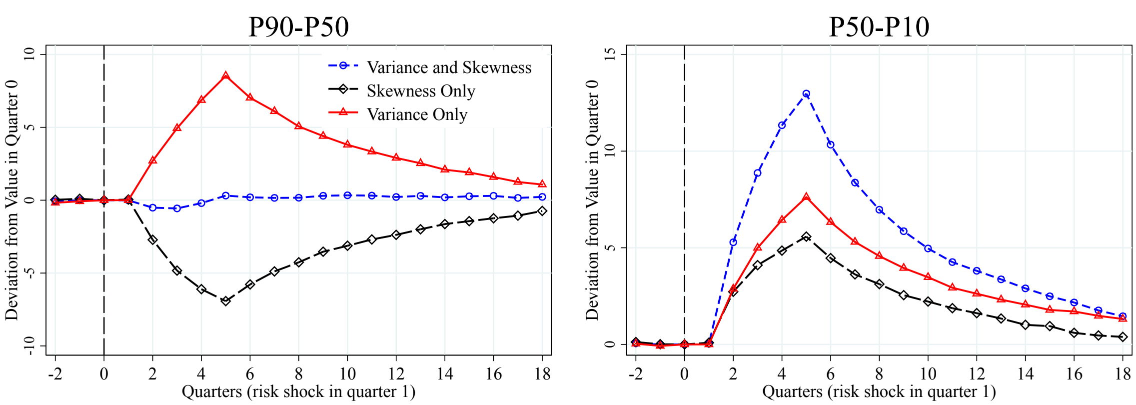
(a) Aggregate Productivity Shock does not Affect Dispersion or Skewness of Sales Growth




(b) Skewness of Employment and Sales Growth

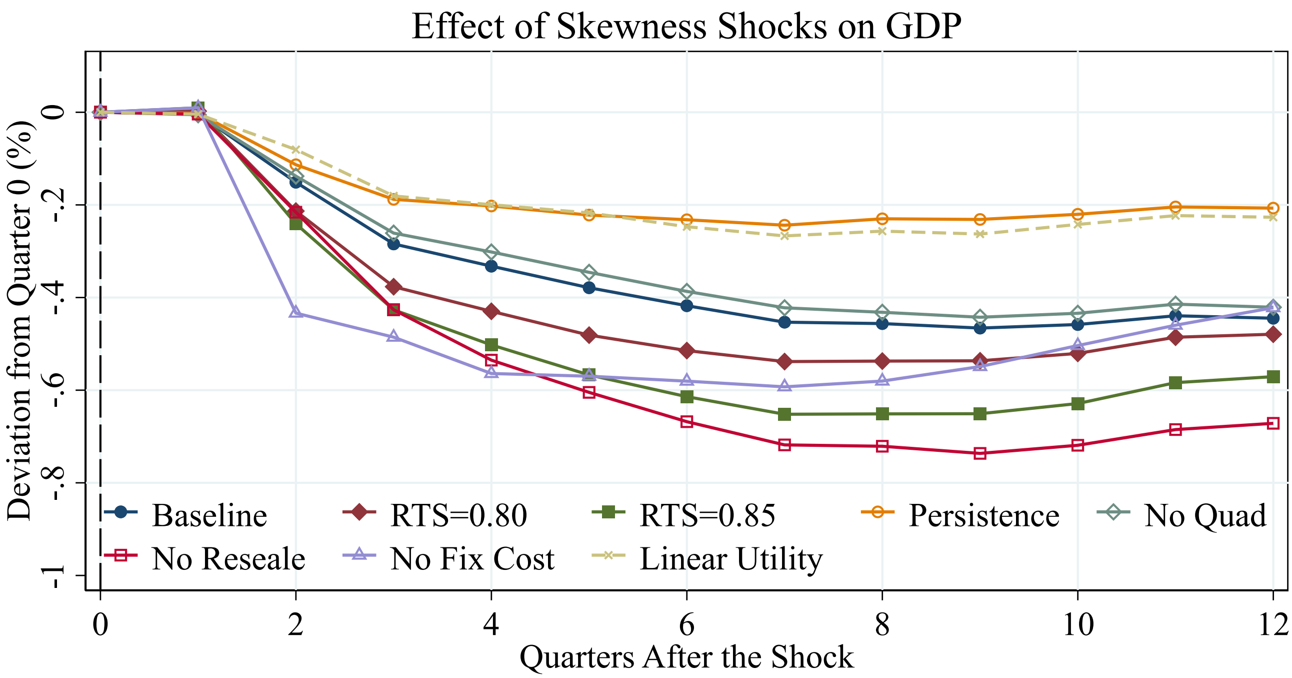


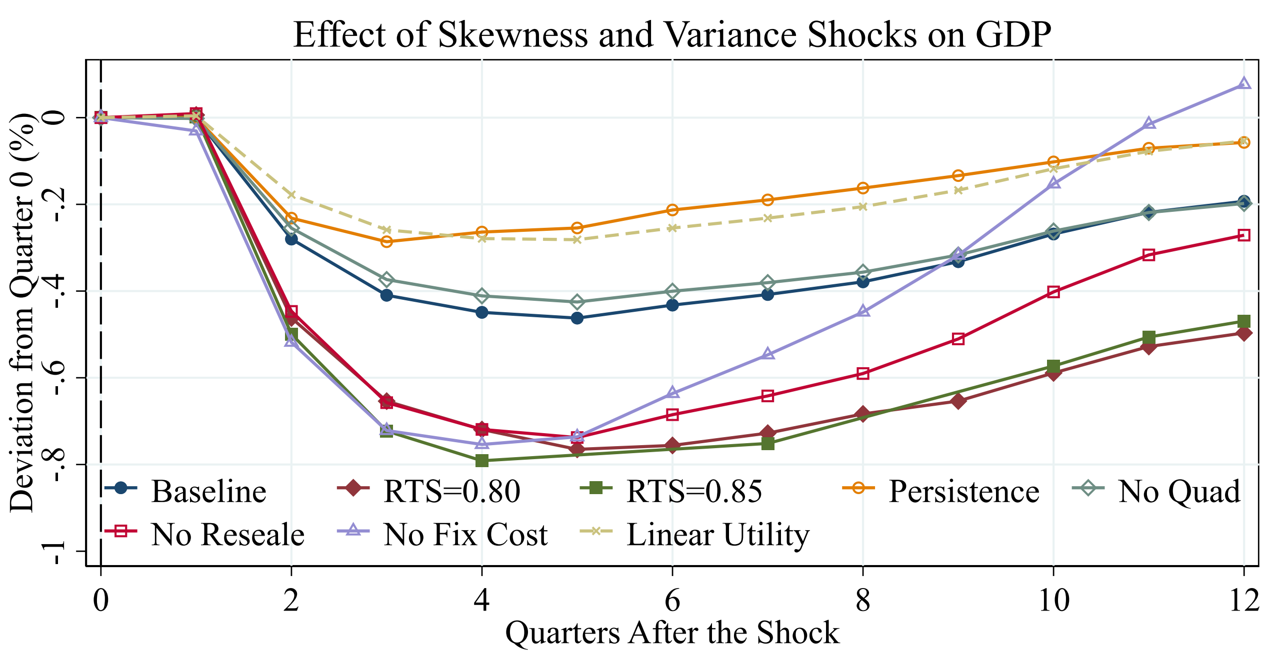

(c) Right and Left Tail Dispersion of Sales Growth




Note: Figure D.4 shows different model-generated moments of the sales growth and employment growth distribution. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. In each simulation, we assume that the economy is in the low-risk state for 150 periods. We then impose a drop in the skewness of firms' shocks in quarter 151, allowing normal evolution of the economy afterwards. We plot the deviation of each macroeconomic aggregate from its value in quarter 0. The red line with triangles trances the impact of an increase in the variance of firms' shocks; the black line with diamonds trances the impact of a drop in the skewness of firms' shocks; the blue line with circles trances the joint impact of an increase in variance and a decrease in skewness.

Figure D.5 – Macroeconomic Effect of a Shock to Skewness and Variance




Note: Figure D.5 shows the effect of a decline in the skewness of firm idiosyncratic productivity. The plot is based on independent simulations of 1,000 economies of 300-quarter length. In each simulation, we assume that the economy is in the low-risk state for 150 periods. We then impose a drop in the skewness of firms' shocks in quarter 151, allowing normal evolution of the economy afterwards. We plot the log percentage deviation of each macroeconomic aggregate from its value in quarter 0. Top panel shows the effects of output, whereas the bottom panel shows the impact on labor, investment in capital, consumption, and investment in the risk-free asset.