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University of Minnesota
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Syllabus for
Computational Methods for Heterogeneous Agent Models
ECON 8185

The objective of this course is to teach computational methods for solving heterogeneous-agent models in macro, labor, IO, trade, and other fields. The methods we will cover in this class will not only allow you to solve the most advanced, state-of-the-art quantitative models used in economics today but also allow you to do so efficiently and accurately. We will learn different methods that differ in their speed, accuracy, as well as their suitability for parallelization. The only way to learn computation is by doing it, so I will ask you to implement each method we learn on the computer. Be ready for lots of exciting assignments.

Course Requirements

Although not required, familiarity with Fortran, C/C++, or Julia, and knowledge of MATLAB or Python will be helpful. If you are not familiar with any programming language, be prepared to work harder than others to catch up, especially in the first half of the course. A solid understanding of first year macro and micro is required too.

To get credit for this course you will need to complete **all** the homework assignments that will be distributed each week. All assignments will need to be submitted electronically. I will describe how to do this in class. Each submission needs to include all the computer code you wrote, the relevant output in a clear and understandable format, and a short report written as an executive summary of your answers/findings.

Contact information

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Textbooks and Reading Materials

The lectures will mainly draw on my lecture notes. I am writing a PhD level textbook on computational and empirical methods for heterogeneous-agent models, and half of the book covers topics included in this course. I will distribute chapters before each lecture.

Here is a list of some books that are complementary to my notes. They contain detailed and authoritative treatments of the subjects we will study in this course.

Some useful books on numerical methods:

- ***A Classic****: Numerical Recipes in Fortran 77: The Art of Scientific Computing*, Second Edition, by William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, 1992. (There

is a subsequent book entitled Numerical Recipes in Fortran 90, which has more up to date Fortran codes. However, the Fortran 77 version contains in-depth discussions of the numerical methods, which are not repeated in the second book. This is one of the most well-written numerical methods books you will find. (Buying the book will be one of the best investments you can make.)

- *Introduction to Numerical Analysis*, by J. Stoer and R. Bulirsch, 3rd Edition, Springer, 2010.

- *Numerical Methods in Economics*, by Kenneth L. Judd, MIT Press, 1998.

- *A First Course in Numerical Analysis*, by Ralston and Rabinowitz, Dover, 1965, 1978.

- A very useful resource: Quantitative Economics with Julia: https://julia.quantecon.org/intro.html (by Perla, Sargent, Stachurski)

TENTATIVE SCHEDULE

WEEK 1: Introduction and Dynamic Programming

- General considerations in numerical analysis: convergence, roundoff error, truncation error (ill conditioned matrices), portability of programs across platforms.
- Numerical dynamic programming:

    - Value Function iteration (standard, policy improvement algorithm)
    - Acceleration via MacQueen-Porteus bounds
    - The Endogenous Grid Method
    - EGM with fixed costs, kinks, etc.

Reading Materials:

- Chapter 14 from my Manuscript (Solution Methods for Dynamic Programming)
- Chapters 1, 5.7, and 9 in Numerical Recipes; Chapters 1, 2, 5, and 7.7 in Judd.

WEEK 2: Numerical Differentiation, Local and Global Optimization

- Numerical differentiation; Root-finding in one or more dimensions.
- Minimization:

    - **Local methods**: Golden section search, Brent's method with or without derivatives, Nelder-Mead algorithm, Newton-Raphson, Derivative-Free Nonlinear Least Squares (DFNLS) algorithm
    - **Global methods**: In-depth: TikTak algorithm, sequential and parallelized versions. Review of NLOPT algorithms

Reading Materials:

- Chapter 10 in Numerical Recipes; Chapter 4 in Judd.
- Christiano, L. J., and J. Fisher (2000), "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints," Journal of Economic Dynamics and Control 24, 1179–1232.
- Arnaud, Guvenen, Kleineberg (2019): "Benchmarking Global Optimizers", NBER Working Paper, No 26340.

WEEK 3: Interpolation and Integration

- Interpolation and approximation of functions: cubic splines, polynomial interpolation (Chebyshev and others).
    a. The CES trick to reduce curvature
    b. Shape preserving splines
- Numerical integration: (i) Romberg integration and Richardson's deferred approach to the limit, (ii) Gaussian quadrature, and Gauss-Kronrod (iii) automatic integrators: QUADPACK (QAGS, Clenshaw-Curtis, etc.)

Reading Materials:

- From my manuscript: Chapter 7 (interpolation), Chapter 8 (integration).
- Chapters 3 and 4 in Numerical Recipes.

WEEK 4: Applying What We Learned So Far

- Discretization methods for a continuous shock process (Tauchen (1986), Tauchen and Hussey (1991), Rouwenhorst (1995), and several new variants of this last method).

- Putting Everything Together: Solving an income fluctuation problem.
    - Benchmarking different algorithms for speed and accuracy.
    - How to ensure the accuracy of solution?

Reading Materials:

- From my book:
    - Chapter 9 (discretization)
    - Chapter 15 (Putting everything together)

WEEK 5: General Equilibrium *Without* Aggregate Shocks.

Two different frameworks:

- The Aiyagari (1994) Model: Analyzing the model.

- Solving the model: Solution via simulation (time series). Solution in state space.
- Bewley/Hugget model. Production vs Exchange economy models

- Power Law Models:
  - Brief review: Models of wealth inequality that yield a Pareto tail: Benhabit et al (2011, 2012, 2013), Gabaix et al (2016), Jones and Kim (2018)
  - How to solve these models?
    - Taming the Pareto tail
    - Simulation strategies
    - A better approach: Analytical methods

Reading Materials:

- Aiyagari, S.R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109, 659–684. (Also take a look at the 1993 working paper version, which contains more details about the computation).
- I will distribute slides.

WEEK 6: General Equilibrium *With* Aggregate Shocks

- Krusell and Smith (1998):
  - The classic K-S algorithm. Implementation. Checking for accuracy.
  - A different approach: Reiter's algorithm in state space.
  - A third approach: Tracking history of shocks (Veracierto-Lustig algorithm)

Reading Materials:

Den Haan, W.J. (1997), "Solving Dynamic Models with Aggregate Shocks and Heterogeneous Agents," *Macroeconomic Dynamics* 1, 355–386.

Huggett, M. (1993), "The Risk-Free Rate in Heterogeneous-Agents, Incomplete Markets Economies," *Journal of Economic Dynamics and Control* 17, 953–969.

Krusell, P. and A.A. Smith, Jr. (1998), "Income and Wealth Heterogeneity in the Macroeconomy," Journal of Political Economy 106, 867–896.

Rios-Rull, J.V. (1995): "Models with Heterogeneous Agents," Chapter 4 in *Frontiers of Business Cycle Research*, ed. Thomas Cooley, Princeton University Press.

Rios-Rull, J.V. (1999): "Computation of Equilibria in Heterogeneous-Agent Models," in *Computational Methods for the Study of Dynamic Economies.*

WEEK 7: GE Models with Bells and Whistles

- Reiter's (2009) method for solving Krusell-Smith type models.

- Application to Khan-Thomas (ECMA, 2008) model of firm dynamics.

The following topics will be selected time permitting:

- Problems that introduce kinks and/or non-concavities:
  - Models with Discrete Choices (marriage/divorce, occupational choice, education decision, among others)
  - Models with Fixed Costs (housing market, financial transactions with fixed costs)

- Extensions of EGM to problems with kinks and/or with discrete and continuous choices

- Models with multiple assets with prices solved through market clearing (Krusell and Smith (1997), Storesletten, Telmer, and Yaron (2006), Guvenen (2009): Two asset problem in general equilibrium, non-trivial market clearing.)

Reading Materials:

Krusell, P. and A.A. Smith, Jr. (1997), "Income and Wealth Heterogeneity, Portfolio Selection, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, 1, 387–422.

Reiter, Michael (2009): "Solving heterogeneous-agent models by projection and perturbation", *Journal of Economic Dynamics and Control*, 33(3), 649-665.

Winberry, Thomas (2018): "A method for solving and estimating heterogeneous agent macro models", *Quantitative Economics*, 9(3), 1123-1151.

Terry, Stephen (2017): "Alternative Methods for Solving Heterogeneous Firm Models," *Journal of Money, Credit, and Banking.*

Guvenen, M.F. (2009) "A Parsimonious Macroeconomic Model for Asset Pricing" *Econometric*a.

Fella, G. (2014): "A generalized endogenous grid method for non-smooth and non-concave problems," *Review of Economic Dynamics*, 17(2), 329‑344.

Iskhakov, Jorgense, Rust, and Schjerning (2017): The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks, *Quantitative Economics*, Vol 8, No 2.

Aubik Khan and Julia Thomas (2008): "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 2008.

Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018): Really Uncertain Business Cycles, Econometrica, Vol 86, No 3.

WEEK 8 (Time Permitting): Global Optimization (Coverage in Week 2 will be expanded if time permits):

- Quasi-Random Numbers: The concept of low discrepancy sequences, Sobol', Halton, and so on. A Simple Stochastic Global Optimization Algorithm; Parallelizing the algorithm without knowing parallel programming.Refinements: Clustering and Pre-testing.

- NLOPT package; Controlled Random Search and other algorithms.

Reading Materials:

Antoine Arnaud, Fatih Guvenen, and Tatjana Kleineberg (2022): "Benchmarking Global Optimizer," Working Paper, University of Minnesota.

Zhang, Conn, Scheinberg (2009): "A Derivative-Free Algorithm for Least Squares Optimization", *SIAM Journal.*