UNDERSTANDING THE EVOLUTION OF THE US WAGE DISTRIBUTION: A THEORETICAL ANALYSIS

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Abstract
In this paper, we propose an analytically tractable overlapping-generations model of human capital accumulation and study its implications for the evolution of the US wage distribution from 1970 to 2000. The key feature of the model, and the only source of heterogeneity, is that individuals differ in their ability to accumulate human capital. Therefore, wage inequality results only from differences in human capital accumulation. We examine the response of this model to skill-biased technical change (SBTC) theoretically. We show that in response to SBTC, the model generates behavior consistent with some prominent trends observed in the US data including (i) a rise in overall wage inequality both in the short run and long run, (ii) an initial fall in the education premium followed by a strong recovery, leading to a higher premium in the long run, (iii) the fact that most of this fall and rise takes place among younger workers, (iv) a rise in within-group inequality, (v) stagnation in median wage growth (and a slowdown in aggregate labor productivity), and (vi) a rise in consumption inequality that is much smaller than the rise in wage inequality. These results suggest that the heterogeneity in the ability to accumulate human capital is an important feature for understanding the effects of SBTC and interpreting the transformation of the US labor markets since the 1970s. (JEL: E21, E24, J24, J31)

1. Introduction

The human capital model (Becker 1964; Ben-Porath 1967) has been one of the workhorses in labor economics over the last 40 years. It has been extensively used to understand such issues as educational attainment, on-the-job training, and wage

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growth over the life cycle, among others. It is then perhaps surprising that, with few exceptions, this model has not been applied to study the significant changes observed in the US labor markets since the early 1970s. The goal of this paper is to close this gap: we propose a particular human capital model and study its theoretical implications for the evolution of the wage distribution during this period. Specifically, this paper focuses on the following three dimensions of the changes in the wage distribution.[1]

1. The stagnation of median wages and the slowdown in labor productivity from about 1973 to 1995 (i.e. changes in the *first* moment of the wage distribution).
2. The substantial changes in overall, between-group, and within-group wage inequality during this period (i.e. changes in the *second* moment of the wage distribution).
3. The relatively small rise in consumption inequality despite the large rise in wage inequality (i.e. changes in *lifetime* wage income distribution).

Among these trends, perhaps the most puzzling has been the joint behavior of overall wage inequality and between-group inequality (i.e. the education premium) and, in particular, their movement in *opposite* directions during the 1970s. Juhn et al. (1993) have documented these patterns and stated: “The rise in within-group inequality preceded the increase in returns to observables by over a decade. On the basis of this difference in timing, it seems clear to us that there are at least two unique dimensions of skill (education and skill differences within an education group) that receive unique prices in the labor market” (p. 429). They then added: “Our conclusion is that the general rise in inequality and the rise in education premium are actually distinct economic phenomena” (p. 412). This widely accepted conclusion has led most of the subsequent literature to search for separate driving forces and mechanisms to explain each phenomenon. In contrast, our paper proposes a mechanism that simultaneously generates a monotonic rise in overall inequality and a nonmonotonic change in the education premium even though the model has a single driving force. This is one of the main contributions of the present paper.

Here are the basic features of our model. Individuals begin life with a fixed endowment of *raw labor* (i.e. strength, health, etc.) and are able to accumulate *human capital* (skills, knowledge, etc.) over the life cycle. Raw labor and human capital earn separate wages in the labor market, and each individual supplies both of these factors of production at competitively determined prices (wages). Investment in human capital takes place on the job unless it exceeds a certain fraction of an individual’s time endowment, in which case it is interpreted as *schooling*. Individuals who invest in excess of this threshold for a specified number of years become college graduates. We assume that skills are general (i.e. not firm-specific) and that labor markets are competitive. As a result, the cost of human capital investment will be completely borne by the workers, and firms will adjust the hourly wage rate downward by the fraction

1. For extensive documentation of these trends, see Katz and Murphy (1992), Juhn et al. (1993), Card and Lemieux (2001), Acemoglu (2002), Attanasio et al. (2004), Autor et al. (2005, 2008), and Krueger and Perri (2006).

of time invested on the job. Thus, the cost of human capital investment is the earnings forgone by individuals as they learn new skills.

This framework differs from the standard Ben-Porath model in mainly two ways. The first new feature is the introduction of raw labor as a factor of production, which is motivated by recent empirical evidence. For example, Rendall (2008) conducts a factor analysis of roughly 12,000 occupations as classified by the US Dictionary of Occupational Titles (DOT) and shows that one can reduce the nearly 40 characteristics by which occupations differ to essentially three common factors. After examining how these factors vary across occupations, Rendall interprets them as cognitive ability, physical ability, and motor coordination (i.e. dexterity). Vijverberg and Hartog (2005) and Ingram and Neuman (2006) reach similar conclusions using different revisions of the DOT. Given that these studies attribute to physical skills a key role in understanding the production process of different occupations, it would be hard to justify a model in which a worker's productivity is based solely on human capital. Therefore, in our framework, raw labor captures the combination of physical ability and motor skills, whereas human capital corresponds to cognitive abilities. Furthermore, as we show in Section 3.7, this modification allows a well-defined notion of *returns to skill*, which does not exist in the standard Ben-Porath model but is clearly crucial for studying *skill-biased* technical change. An advantage of the specification we propose is that it retains the analytical tractability of the Ben-Porath framework, which enables us to establish our results theoretically.

Second, we allow individuals to differ in their ability to accumulate human capital. As a result, individuals differ systematically in the amount of investment they undertake and, hence, in the growth rate of their wages over the life cycle. This assumption is consistent with empirical evidence from panel data on individual wages; see Guvenen (2007, 2009), Huggett et al. (2007), and the references therein. Thus, in our model, wage inequality results from the systematic *fanning out* of wage profiles over the life cycle.

The demand side of the model consists of a linear production technology that takes raw labor and human capital as inputs. The driving force behind the nonstationary changes during this period is skill-biased technical change (SBTC) that occurs starting in the early 1970s. A key difference of our model is that it does not equate *skill* with education as was often done in previous studies. We instead interpret skill more broadly as human capital, and we view SBTC as a change that raises the productivity of human capital relative to raw labor. This different perspective has important consequences. To see this, note that in our model all workers have some amount of human capital (which varies by ability and age) and raw labor (which is the same for all). Therefore, SBTC affects wages not only *between* education groups (because of differences in average human capital levels), but also *within* each group, depending on the ability and age of each individual. Moreover, education is not a separate skill with its own price but is merely a noisy indicator of an individual's ability to learn, which in turn is an indicator of how strongly he responds to SBTC. Therefore, another contribution of this paper is to propose a framework in which between- and within-group inequality can be studied jointly.

The linear production function allows us to solve the model in closed form, derive explicit expressions for the moments of the wage and consumption distributions, and establish our results theoretically. Yet in addition to providing this analytical convenience, the linear form plays another important role. With *imperfect* substitution, the college premium would be negatively related to the relative supply of college graduates. Several authors have emphasized this link to argue that the fall in the college premium during the 1970s resulted from a rapid increase in the supply of college-educated workers (Katz and Murphy 1992; Juhn et al. 1993). When the production function is linear, however, this link is broken. We therefore use this linear technology to highlight a different mechanism and to show that our results—and especially the nonmonotonic behavior of the college premium—do not rely on the relative supply channel emphasized in earlier work. In fact, probably both channels are operational and complementary to each other.

We first examine the behavior of average wages in response to SBTC. Under a fairly mild assumption, the model generates stagnation in average wages (and a slowdown in labor productivity) in the short run after SBTC and a rise in the long run. The mechanism behind this result can be explained as follows. Because SBTC raises the returns to human capital at all future dates, it leads to a permanent increase in investment rates since individuals are forward looking. Although this higher investment results in an immediate increase in costs (in the form of forgone earnings), its benefits are realized gradually as the total stock of human capital slowly increases. Consequently, observed wages fall in the short run (owing to increased investment on the job) and inherit the gradual growth of human capital stock thereafter.

Second, a closely related mechanism generates the nonmonotonic behavior of the college premium during SBTC. Because college graduates have higher learning ability than those with lower education, their investment increases more in response to SBTC. This *differential* increase in immediate costs (i.e. forgone earnings) results in a fall in their relative wages in the short run. In the long run, however, this higher investment yields a larger increase in their human capital stock, which leads to a higher college premium. Third, it is also easy to see that the described mechanism will affect younger workers—who have a longer horizon and thus expect larger benefits from investing—more than older ones, resulting in a more pronounced decline in the college premium among younger workers, consistent with empirical evidence (Katz and Murphy 1992; Card and Lemieux 2001). Fourth, despite the fall in the college premium in the short run, it can be shown that overall wage inequality *rises* in the model during the same time (Proposition 5). Taken together, the second and fourth results show that this model is consistent with the joint behavior of overall- and between-group inequality observed in the US data described at the outset. As mentioned previously, this is a novel result in the literature.

Fifth, the rise in lifetime income inequality in the model is significantly smaller than the rise in wage inequality. In the model, a high price of human capital generates larger cross-sectional wage inequality because of a *fanning out* of wage profiles (see Figure 1 in Section 2.2). However, note that those individuals who experience a large increase in their wages later in life are exactly those who make larger investments and accept lower wages early on. In calculating lifetime income, future gains are discounted more than early losses, and so the rise in lifetime inequality—which, in this model, equals consumption inequality—remains small. Therefore, the model offers a new mechanism that rationalizes a small change in consumption inequality by the large increase in wage inequality.

Finally, it is useful to provide a sense about the quantitative implications of this model model even while bearing its highly stylized nature. Toward this end, we simulate a calibrated version of the baseline framework (Section 4) and find that it produces plausible behavior that is consistent with the US data. In a companion paper (Guvenen and Kuruscu 2010), we go further and allow for Bayesian learning about future skill prices, allow imperfect substitution in the production function, introduce uncertainty, and so on. The quantitative analysis in the companion paper shows that the main mechanisms highlighted in this paper continue to play a central role and that the main conclusions of this paper carry over to more general cases. The analysis in Section 4 of this paper focuses on some important empirical facts not studied in the companion paper, such as the behavior of the within-group inequality in the short run and the separate evolutions of top-end and lower-end wage inequality.

There is a vast literature on the empirical trends that motivate this paper; a short list of these papers is given in footnote 1. Heckman et al. (1998) use the standard Ben-Porath framework to examine quantitatively the implications of SBTC for some of the inequality trends mentioned previously. Our paper differs from theirs in several important respects. First, in our model a permanent increase in the *level* of the price of human capital results in a permanent increase in investment, whereas in the Ben-Porath model only changes in the *growth rate* of skill prices affect investment permanently. Therefore, in our model, all measures of inequality *increase* between the short run after SBTC and the long run, whereas many of them—overall inequality, college premium, within-group inequality—*fall* in Heckman et al. (1998). Second, in addition to wage inequality, we also analyze the behavior of average wages after SBTC (i.e., the productivity slowdown puzzle) as well as the changes in consumption inequality, which are not studied by these authors. Finally, one contribution of our paper is to propose a framework for studying human capital accumulation with SBTC that is analytically tractable. As a result, we are able to solve the model in closed form and establish all our results theoretically.[2]

Several other papers have proposed theoretical models in which the rapid increase in skill demand was a driving force for rising wage inequality. Important examples include Hornstein and Krusell (1996), Galor and Tsiddon (1997), Greenwood and Yorukoglu (1997), Caselli (1999), and Violante (2002). Greenwood and Yorukoglu (1997) emphasize the role of skill in facilitating the adoption of new technologies. They argue that the advent of computer technologies in the 1970s presented such a

2. Furthermore, the behavior of within-group inequality in Heckman et al. (1998) is inconsistent with the data. For example, wage inequality among college graduates jumps up right after SBTC and then falls monotonically, whereas the wage inequality among high-school graduates falls first and then increases monotonically (see their Figure 13). As we will discuss later, these findings are at odds with the data.

change, which increased the wages of skilled workers and resulted in a productivity slowdown due to the time it takes to utilize the new technologies effectively. Hornstein and Krusell (1996) make a similar observation but add that the acceleration in quality improvements during this period exacerbated measurement problems, further reducing measured productivity growth. Caselli (1999) studies a model where differences in innate ability and newer technologies that are more costly to learn than existing ones result in a rising skill premium. Violante (2002) develops a model of within-group inequality in which vintage-specific skills, embodied technological acceleration, and labor market frictions combine to generate rising inequality. These papers share the feature that technical change is *embodied* in new machines, but in our paper, such change is *disembodied*. Both types of changes have arguably been taking place during this period, so the mechanisms emphasized in these papers are complementary to ours.[3]

Some authors have proposed explanations for the (nonmonotonic) behavior of the college premium during this period. Katz and Murphy (1992) show that a simple supply–demand framework provides a good fit to the observed behavior of the college premium. Krusell et al. (2000) show that an increased demand for skills can result if capital and skills are complementary in the production function and if technical change is investment-specific. Acemoglu (1998) proposes a model that also endogenizes the demand for skill: essentially, a large rise in the supply of college-educated workers causes firms to direct their innovations to take advantage of this supply, creating an endogenous skill bias in technological progress. He also shows that an extension of this model can be consistent with the joint behavior of between- and within-group inequality. To our knowledge, Acemoglu (1998) is the only paper apart from our model that generates this result.[4]

Finally, Guvenen et al. (2009) build on the framework introduced in this paper by explicitly modeling labor market institutions and allowing for idiosyncratic shocks and endogenous labor supply in order to understand the role of tax policy in wage inequality. They show that the interaction of endogenous human capital investment with differences in the progressivity of the labor income tax schedule between the United States and Continental European countries can quantitatively explain a significant part of the observed inequality gap between these two regions as well as the widening of this inequality gap since the 1970s.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the theoretical analysis and establishes the results already described. Section 4 presents some illustrative simulations, and Section 5 concludes.

3. In the working paper version (Guvenen and Kuruscu 2007, Section 4), we study a variant of the present framework with *embodied* technical change.

4. Gould et al. (2001) suggest, but do not develop, an extension of their model that they conjecture could generate the different behaviors of between- and within-group inequality but with two *separate* driving forces.

2. A Baseline Model

The economy consists of overlapping generations of individuals who live for S years. Individuals begin life with an endowment of *raw labor* (i.e. strength, health, etc.), which is the same across individuals and *constant* over the life cycle, and are able to accumulate *human capital* (skills, knowledge, etc.) over the life cycle. There is a continuum of individuals in every cohort, indexed by $j \in [0, 1]$, who differ in their ability to accumulate human capital, denoted by $\tilde{A}_j$ (i.e. their *type*). This is the only source of heterogeneity in the model.

Each individual has one unit of time endowment in each period that can be allocated between producing output and accumulating human capital. Let l denote raw labor and let h_{js} denote the human capital of an s-year-old individual of type j. We assume that raw labor and human capital earn separate wages in the labor market and that each individual supplies both of these factors of production at competitively determined wage rates. Therefore, the *potential income* of an individual—that is, the income he would earn if he spent all his time producing for his employer—is given by $P_L l + P_H h_{js}$, where P_L and P_H are the rental prices of raw labor and human capital, respectively. (For clarity of notation, we suppress the dependence of variables on time except when we want to emphasize time variation.)

Following the standard interpretation of the Ben-Porath (1967) model, we assume that investment in human capital takes place on the job unless it exceeds a fraction $i^* \in (0, 1]$ of an individual's time, in which case the investment is interpreted as *schooling*. We assume that skills are general (i.e. not firm-specific) and that labor markets are competitive. As a result, the cost of human capital investment will be completely borne by workers, and firms will adjust the hourly wage rate downward by the fraction of time invested on the job (Becker 1964). Then, the observed wage income of an individual is given by

$$w_{js} = \left[P_L l + P_H h_{js}\right](1 - i_{js}) = \underbrace{\left[P_L l + P_H h_{js}\right]}_{\text{Potential earnings}} - \underbrace{\left[P_L l + P_H h_{js}\right] \times i_{js}}_{\text{Cost of investment}}, \quad (1)$$

where i_{js} is the fraction of time spent on human capital investment, henceforth referred to as *investment time*. Thus, wage income can be written as potential earnings minus the *cost of investment*, which is simply the earnings forgone while individuals are learning new skills. Since labor supply is inelastic (i.e. conditional on working, all workers supply one unit of time per period), it follows that w_{js} is also the observed (hourly) wage rate.

Individuals begin their life with zero human capital, $h_{j,0} = 0$, and accumulate human capital according to the following technology:

$$h_{j,s+1} = h_{js} + Q_{js}. \quad (2)$$

Here Q_{js} is the newly produced human capital, which will be referred to simply as *investment* in the rest of the paper and should not be confused with investment time (i_{js}). New human capital is produced by combining the existing stocks of raw labor and human capital with the available investment time according to

$$Q_{js} = \tilde{A}_j((\theta_L l + \theta_H h_{js}) i_{js})^{\alpha}. \tag{3}$$

The key parameter in this specification is $\tilde{A}_j$, which determines the productivity of learning. Because of the heterogeneity in $\tilde{A}_j$, individuals will differ systematically in the amount of investment they undertake and hence in the growth rate of their wages over the life cycle. Another important parameter is $\alpha \in [0, 1]$, which determines the degree of diminishing marginal returns in the human capital production function. A low value of α implies higher diminishing returns, in which case it is optimal to spread out investment over time. In contrast, when α is high, the marginal return on investment does not fall quickly, so investment becomes bunched over time. In the extreme case when $\alpha = 1$, individuals will spend either all their time on investment ($i_{js} = 1$) or none at all in a given period. Finally, the appearance of raw labor in equation (3) captures the plausible idea that an individual's physical capacity (health, strength, patience, etc.) affects the learning process. The parameters θ_L and θ_H determine the relative contributions of each factor to human capital accumulation and could be time-varying as well.

The Individual's Dynamic Problem. We assume that individuals can borrow and lend at a constant interest rate (denoted by r), which implies that markets are complete. As is well known, in this case the consumption–savings and income maximization decisions can be disentangled from each other. Hence, for the purposes of analyzing human capital investment, we concentrate on the lifetime income maximization problem. Individuals solve

$$\max_{\{i_{js}\}_{s=1}^{S}} \left[\sum_{s=1}^{S} \left(\frac{1}{1+r} \right)^{s-1} \left[P_L l + P_H h_{js} \right] (1 - i_{js}) \right]$$

subject to (2), (3), and $h_{j,0} = 0$. It should be emphasized that this formulation does not require risk neutrality; it requires only that markets be complete.

2.1. Aggregate Production Technology

The aggregate factors used in production at a given moment in time are defined as

$$L^{\text{net}} = \sum_{s=1}^{S} \mu(s) \int_j l \left(1 - i_{js}\right) dj \quad \text{and} \quad H^{\text{net}} = \sum_{s=1}^{S} \mu(s) \int_j h_{js} \left(1 - i_{js}\right) dj,$$

where $\mu(s)$ is the (discrete) measure of s-year-old individuals and the sums are thus taken over the distribution of individuals of all types and ages.[5] The superscript "net" indicates that these variables measure the actual amounts of each factor *used in*

5. For the population structure assumed so far, $\mu(s) = 1/S$.

production (i.e. net of the time allocated to human capital investment) in order to distinguish them from the *total stocks* of these factors available in the economy, which are defined later. The aggregate firm uses these two inputs to produce a single good, denoted by Y, according to

$$Y = Z\left(\theta_L L^{\text{net}} + \theta_H H^{\text{net}}\right), \tag{4}$$

where Z is the total factor productivity (TFP). For simplicity, we assume that capital is not used in production. (Note that raw labor and human capital enter the aggregate production function and human capital production in a symmetric manner and with the same productivity parameters; compare (4) with (3).) This assumption allows us to solve the model in closed form. Furthermore, as we show in Guvenen and Kuruscu (2010), it also produces quantitative implications that are quite plausible. The firm solves a static problem by hiring factors from households every period to maximize its profit: $Y - P_L L^{\text{net}} - P_H H^{\text{net}}$. As a result, factor prices are given by the marginal products: $P_H = \partial Y/\partial H^{\text{net}} = \theta_H$ and $P_L = \partial Y/\partial L^{\text{net}} = \theta_L$.

It is useful to compare this production structure to that assumed in some of the previous literature. In those papers, the production technology is typically taken to be a constant elasticity of substitution (CES) function: $Z[(\theta_L L)^\rho + (\theta_H H)^\rho]^{1/\rho}$, where now H and L denote the total *work hours of college and high-school workers*, respectively. Therefore, in such models, a change in θ_H/θ_L due to SBTC has the same effect on all individuals *within* a given education group. In this paper, however, we interpret skill more broadly as human capital and do not equate it to education. Since all workers in this model have some amount of human capital (which varies by ability and age) and raw labor (which is the same for all), SBTC not only changes wages *between* education groups (because of differences in average human capital levels) but also affects wages *within* each group differently, depending on the ability and age of each individual. Hence, this model allows us to study both between- and within-group inequality simultaneously.

A second implication of the production function assumed in these studies is that the education premium is given by $P_H/P_L = (\theta_H/\theta_L)^\rho (H/L)^{\rho-1}$, and is therefore decreasing in the relative supply of college graduates. Several authors have emphasized this link to argue that the fall in the college premium during the 1970s resulted from the rapid increase in the supply of college-educated workers (Katz and Murphy 1992; Juhn et al. 1993). Yet notice that this link is broken when the production function is linear: $P_H/P_L = \theta_H/\theta_L$. Hence, we use this linear production technology to show that the nonmonotonic behavior of the college premium in our framework is not driven by the relative supply channel emphasized in earlier research.

2.2. *Analyzing the Individual's Problem*

We now rewrite the problem to simplify exposition. Using equation (3), the opportunity cost of investing an amount Q_{js} can be written as $C_j(Q_{js}) \equiv (\theta_L l + \theta_H h_{js}) i_{js} = (Q_{js}/\tilde{A}_j)^{1/\alpha}$. With this transformation, the problem of an individual can be

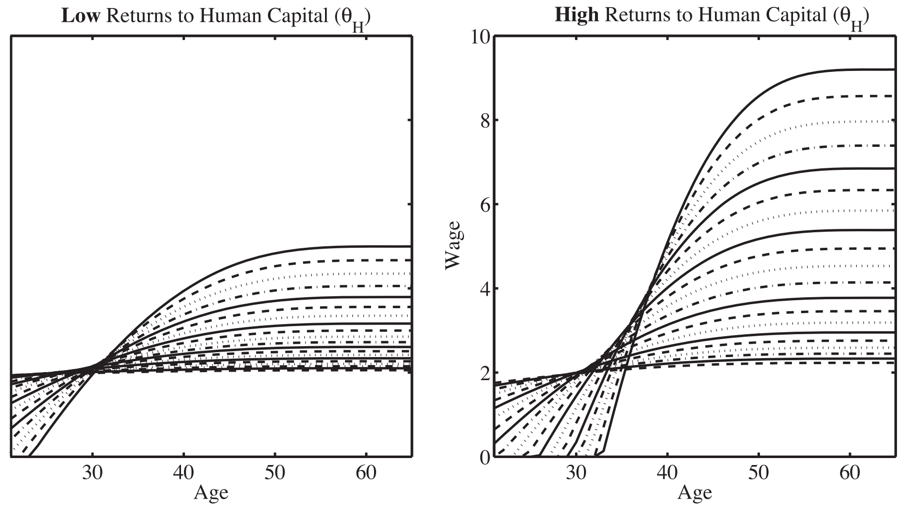

FIGURE 1. Effect of the price of human capital on life-cycle wage profiles.

written as

$$\max_{\{Q_{js}\}_{s=1}^{S}} \left[\sum_{s=1}^{S} \left(\frac{1}{1+r} \right)^{s-1} (\theta_L l + \theta_H h_{js} - C_j(Q_{js})) \right]$$
$$\text{s.t.} \quad h_{j,s+1} = h_{js} + Q_{js} \ \text{ with } h_{j,0} = 0.$$

The optimality condition that determines the amount of investment at time t is

$$C_j'(Q_{js}) = \left\{ \frac{\theta_H(t+1)}{1+r} + \frac{\theta_H(t+2)}{(1+r)^2} + \cdots + \frac{\theta_H(t+S-s-1)}{(1+r)^{S-s-1}} \right\}, \tag{5}$$

where we make explicit the dependence of future prices of human capital on time. The left-hand side of this equation is the marginal cost, and the right-hand side is the marginal benefit (MB) of investment. The latter is equal to the present discounted value of the future stream of wages that is earned by an additional unit of human capital. An important implication of (5) is that an expected increase in the future prices of skill (the sequence $\theta_H(t)$) will have an immediate permanent impact on current investment decisions because of the forward-looking nature of this equation. As we elaborate in Section 3.7, this result is in contrast to the standard Ben-Porath model, where a rise in the price of human capital has no effect on investment behavior.

We now return to analyzing the individual's problem. To illustrate how the model works, consider two economies that differ only in the price of human capital, θ_H and θ_H' with $\theta_H' > \theta_H$. Figure 1 compares the wage profiles of individuals with different ability levels in these two cases. First, note the features common to both cases: workers with high ability invest more than others, accepting lower wages early on in return for higher wages later in life. As a result, wage inequality increases over the life cycle owing to the systematic fanning out of the wage profiles. Workers with ability level above a certain threshold invest full-time early in life (i.e. they attend college) and have zero wage income.

A comparison of these two economies reveals a number of important points that are key to understanding the results of this paper about the long-run effects of SBTC. First, a higher price of human capital induces more investment, where the strength of this response increases with ability. Hence, cross-sectional wage inequality increases because of the fanning out of wage profiles. Observe, however, that lifetime income inequality will not rise as much as cross-sectional wage inequality because those with high wages later on are exactly those who invest more and thus have low wages early in the life cycle. Moreover, since lifetime income is a discounted average of wages over the life cycle, later gains are discounted compared with early losses and so the rise in the lifetime income of high-ability individuals remains modest. We return to this point in Section 3.8.

3. Theoretical Analysis

Although the model just described can be solved in closed form to obtain explicit expressions for individuals' wage rates and consumption choices, the resulting expressions are extremely complicated. In the rest of this paper, we examine statistics related to entire cross sections (e.g. the mean and variance of wage and consumption), which requires aggregating these objects over all cohorts and ability levels. Of course, such an exercise becomes tedious quickly.

To overcome this difficulty, we assume a simplified demographic structure that allows us to establish our main results theoretically. In particular, we specialize to the *perpetual youth* version of the overlapping-generations model as in Blanchard (1985): individuals can potentially live forever ($S = \infty$) but face a constant probability of death $(1 - \delta)$ every period. Under this assumption, s is no longer a state variable in the human capital problem, which simplifies the analysis substantially. We normalize the population size to 1, and we assume that each period a cohort of measure $1 - \delta$ is born to replace the individuals who die. Therefore, the measure of an s-year-old cohort is given by $\mu(s) = (1 - \delta)\delta^{s-1}$. In the rest of the analysis, we restrict our attention to an interior solution; hence we assume that $w_{js} \geq 0$ for all j and s. This assumption allows for analytical tractability. Finally, we assume that $r = 1/(\delta\beta) - 1$, where β is the pure time discount factor. This assumption implies that individuals choose a constant consumption path over the life cycle.

3.1. Characterizing the Steady State before SBTC

In order to examine the effects of skill-biased technical change, we first assume that the economy is in steady state in the period preceding the shock and then characterize how investment, wages, and consumption are determined. In this initial steady state, let $\theta_H(t) = \theta_H$ and $\theta_L(t) = \theta_L$ for all t.

The assumption of constant survival probability simplifies the structure of the model in many ways. First, the optimality condition for investment choice (5) reduces to $C'_j(Q_j) = \theta_H \beta\delta\,(1-\beta\delta)^{-1}$, where the marginal benefit of investment is constant, because the expected life span is now independent of age. Using the functional form for the cost function yields

$$Q_j = A_j \left(\frac{\alpha\delta\beta}{1-\beta\delta} \theta_H \right)^{\alpha/(1-\alpha)}, \tag{6}$$

where $A_j \equiv \tilde{A}_j^{1/(1-\alpha)}$. That Q_j is independent of age implies that the human capital stock at age s is simply $h_{js} = Q_j(s-1)$. The optimal amount of investment time, i_{js}, is given by the total cost of investment divided by potential earnings:[6]

$$i_{js} = \frac{C(Q_j)}{\theta_L l + \theta_H h_{js}} = \frac{\alpha\delta\beta}{1-\delta\beta} \left[\frac{\theta_L l}{\theta_H Q_j} + (s-1) \right]^{-1}. \tag{7}$$

A few intuitive results can be seen from these expressions. These results play an important role in proving the propositions to follow and we summarize them in the following lemma.

LEMMA 1. *In the overlapping-generations model of human capital accumulation described so far, optimal investment choice has the following properties.*

(i) *Individuals with higher ability make larger investments: $dQ_j/dA_j > 0$ (from (6)).*
(ii) *Even though individuals increase their human capital stock by a constant amount Q_j every period, investment time falls with age: $di_{js}/ds < 0$ (from (7)).*
(iii) *For a given age, individuals with higher ability also devote a larger fraction of their time investing in human capital: $di_{js}/dA_j > 0$ (obtained by combining (6) and (7)).*
(iv) *Finally, and most importantly, the increase in investment time in response to SBTC is larger for individuals with higher ability: $d^2 i_{js}/d\theta_H dA_j > 0$.*

Average Wage Rate in the Economy. It is useful to define some new variables in order to express the average wage rate in an easily interpretable form. First, the wage of an s-year-old individual of type j can be written as $w_{js} = \theta_L l + \theta_H(s-1)Q_j - C(Q_j)$. Next, define the *average investment* in the economy,

$$\bar{Q} \equiv \sum_{s=1}^{\infty} \mu(s) \int_j Q_j \, dj = \left(\frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \right)^{\alpha/(1-\alpha)} E[A_j], \tag{8}$$

the corresponding *average cost of investment*,

$$C(\bar{Q}) \equiv \sum_{s=1}^{\infty} \mu(s) \int_j C\left(Q_j\right) \, dj = \frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \bar{Q},$$

6. In what follows, we drop the subscript j from the cost function because optimal investment choice satisfies the equality

$$C_j(Q_j) = \alpha C'_j(Q_j) Q_j = \left(\frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \right) Q_j.$$

Thus, the cost of investment evaluated at the optimal investment level depends on j only through Q_j.

the *average human capital stock*,

$$H\left(\bar{Q}\right) \equiv \sum_{s=1}^{\infty} \mu(s) \int_j h_{js}\, dj = \sum_{s=1}^{\infty} \mu(s)(s-1) \times \int_j Q_j\, dj = \frac{\delta}{1-\delta}\bar{Q}, \tag{9}$$

and, finally, the *average raw labor endowment* in the economy,

$$L \equiv \sum_{s=1}^{\infty} \mu(s) \int_j l\, dj = l.$$

At a given moment in time, $\bar{Q}$ and $C(\bar{Q})$ depend only on the *future* values of θ_H whereas the *stock* of human capital only depends on *past* levels of investment, which in turn are determined by the history of θ_H.[7] Therefore, the former variables will adjust immediately in response to a permanent change in θ_H such as SBTC (making them *jump variables*), whereas $H\left(\bar{Q}\right)$ will adjust only gradually (making it a *stock variable*). This distinction will play a crucial role in the subsequent analysis. Now we can use the expression for w_{js} to calculate the *average wage rate* in the economy as[8]

$$\begin{aligned} \bar{w} &\equiv \sum_{s=1}^{\infty} \mu(s) \int_j w_{js}\, dj = \left[\theta_L l + \theta_H H\left(\bar{Q}\right)\right] - C(\bar{Q}) \\ &= \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H \bar{Q}, \end{aligned} \tag{10}$$

where the last equality is derived by substituting the expressions for $H\left(\bar{Q}\right)$ and $C(\bar{Q})$.

Optimal Consumption Choice. Given the interest rate $r = 1/(\delta\beta) - 1$, the optimal consumption path is constant over the life cycle and is given by the fraction $1 - \delta\beta$ of individuals' lifetime income. Then the *average consumption* in the economy is

$$\bar{c} = \left[\theta_L l + \frac{\delta\beta}{1-\delta\beta}\theta_H \bar{Q}\right] - C(\bar{Q}) = \theta_L l + \left((1-\alpha)\frac{\delta\beta}{1-\delta\beta}\right)\theta_H \bar{Q}. \tag{11}$$

Comparing (10) and (11), it is easy to see that average consumption is less than average wage ($\bar{c} < \bar{w}$) whenever $\beta < 1$. This can be explained as follows. Given that the interest rate equals the reciprocal of the effective discount rate ($\delta\beta$), individuals would like to maintain a constant consumption over their lifetime. But because all individuals have upward-sloping wage profiles, they need to borrow against their future income in order to maintain a constant consumption path. With a positive interest rate, part of aggregate labor income goes toward paying the interest that accrues on borrowed funds (which, for simplicity, is assumed to be borrowed from the rest of the world). As a result, average consumption is less than the average wage in the economy.

7. As a result, the definition of $H\left(\bar{Q}\right)$ in equation (9) is valid only in steady state when all past returns are constant. Similarly, the definitions of $\bar{Q}$ and $C(\bar{Q})$ are valid only when all future returns are constant.

8. Because $H(\bar{Q})$ and L measure (respectively) the total aggregate human capital stock and raw labor in the economy, they include the amount of these factors used to produce human capital. They should not be confused with the net amounts of these factors used in producing output only, H^{net} and L^{net}, defined in Section 2.1.

3.2. Characterizing the Behavior after SBTC

In the theoretical analysis that follows, we consider a one-time permanent increase at time t^* in the price of human capital from θ_H to θ'_H while the price of raw labor, θ_L, remains constant.[9] We analyze the behavior of average wages (and labor productivity), the college premium, and overall wage inequality in both the short run and the long run. For the short-run analysis, we focus our attention to the period immediately after the shock to capture the fact that, in the short run, the human capital stock does not fully adjust, but investment jumps to its new level immediately.

3.3. Slowdown in Labor Productivity (Average Wages)

Labor economists and macroeconomists have documented two closely related trends during the period we study: the stagnation of median wage growth and the slowdown in labor productivity growth; both started with a sharp fall in 1973 and persisted until about 1995. For example, the median real wage increased by 2.2% per year between 1963 and 1973 but actually *fell* by 0.3% per year between 1973 and 1989 (Juhn et al. 1993). Similarly, labor productivity (measured as nonfarm business output per hour) grew by 2.6% per year from 1955 to 1973 but only by 1.45% per year from 1973 to 1995.[10]

To develop the implications of our model, first consider the average wage in the economy in the initial (hence the "I" subscript) steady state:

$$\bar{w}_{\mathrm{I}} \equiv \bar{w}|_{t<t^*} = \left[\theta_L l + \theta_H \times H\left(\bar{Q}\right)\right] - C(\bar{Q}). \tag{12}$$

As noted before, θ_H and $C(\bar{Q})$ increase immediately after SBTC whereas $H\left(\bar{Q}\right)$ adjusts only gradually. Therefore, the average wage immediately after SBTC (in the short run, SR) is given by

$$\bar{w}_{\mathrm{SR}} \equiv \bar{w}|_{t=t^*+\varepsilon} = \left[\theta_L l + \theta'_H \times H\left(\bar{Q}\right)\right] - C(\bar{Q}'), \tag{13}$$

where a prime denotes the value of a variable in the final steady state. The stock of human capital, $H(\bar{Q})$, gradually increases to its new steady-state value $H(\bar{Q}')$, and the average wage in the new steady state is given by

$$\bar{w}_{\mathrm{LR}} \equiv \bar{w}|_{t\to\infty} = \left[\theta_L l + \theta'_H \times H(\bar{Q}')\right] - C(\bar{Q}'). \tag{14}$$

9. Since θ_L remains unchanged and θ_H increases, SBTC entails a true improvement in aggregate productivity in these experiments. An alternative way of modeling SBTC would be to assume that the rise in θ_H is matched by a symmetric fall in θ_L. The results proved in the following sections carry over to this case and some of them become easier to prove. For example, the decline in average wage in the short run would be larger in this case. Similarly, after SBTC, consumption inequality would increase even less than it does under the current formulation. To show that these results do not follow trivially from the decline in θ_L, we assume that θ_L is fixed and θ_H increases.

10. Authors' calculation from Bureau of Labor Statistics data.

Price, Investment, and Quantity Effects. It is instructive to decompose the changes in the average wage after SBTC into three components. We begin by using equations (12) and (13) to write the short-run response of the average wage as

$$\bar{w}_{\mathrm{SR}} - \bar{w}_{\mathrm{I}} = \underbrace{\left[\left(\theta_H' - \theta_H\right) \times H\left(\bar{Q}\right)\right]}_{\text{Price effect } (>0)} + \underbrace{\left[C\left(\bar{Q}\right) - C(\bar{Q}')\right]}_{\text{Investment effect } (<0)}. \tag{15}$$

First, for a fixed stock of human capital, an increase in θ_H *increases* the wage rate. We call this the *price effect*. Second, a higher θ_H also induces more investment, which *reduces* the wage rate by increasing the forgone earnings; we refer to this as the *investment effect*. Therefore, the short-run response of the average wage is entirely determined by the relative strengths of these counteracting forces. In other words, whether or not the average wage falls in the short run depends on whether or not the investment effect dominates the price effect. We will examine the conditions under which this outcome obtains.

Similarly, using equations (13) and (14) yields

$$\bar{w}_{\mathrm{LR}} - \bar{w}_{\mathrm{SR}} = \underbrace{\theta_H' \times \left(H(\bar{Q}') - H\left(\bar{Q}\right)\right)}_{\text{Quantity effect } (>0)}, \tag{16}$$

which shows that the only change between the short run and the long run is the (slow) adjustment of the human capital stock. We call this long-run channel the *quantity effect*. Finally, adding equations (15) and (16) shows that the total effect of SBTC on the average wage ($\bar{w}_{\mathrm{LR}} - \bar{w}_{\mathrm{I}}$) can be written simply as the sum of the price, investment, and quantity effects. We will use analogous decompositions to examine the effect of SBTC on other variables in what follows, although the main idea will be the same as here.

The following condition is needed to establish Propositions 1 and 2. It characterizes the parameter values under which the investment effect dominates the price effect.

CONDITION 1. $\dfrac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} > \dfrac{\delta}{1-\delta}$.

Figure 2 illustrates the parameter combinations that satisfy Condition 1. From equation (15), it is clear that the investment effect would dominate the price effect if either the initial stock of human capital is low (so that the price effect is small), or the response of investment is high. The latter is, in turn, mainly determined by two parameters. First, the response of investment to SBTC is larger when α is high. This is because, as noted earlier, a higher α implies less diminishing marginal returns in human capital production. Consequently, there is little benefit from spreading investment over time (as would be the case if α were low.) Second, for a given (α, δ), a higher β makes the present discounted value of future wages larger, implying a higher benefit from a given increase in the price of human capital; thus, the response of investment to SBTC increases with β (and the corresponding low interest rate). At the same time, the stock of human capital is increasing in the survival probability, so the price effect is more

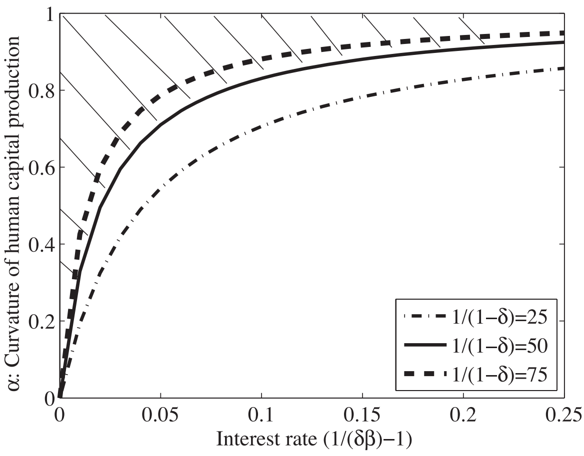

FIGURE 2. Combinations of (α, r) that satisfy Condition 1 (shaded region).

likely to dominate the investment effect when δ is large.[11] The combination of these three forces gives rise to the region of admissible parameters shown in Figure 2. This region contains a fairly wide range of plausible parameter combinations. For example, assuming an expected working life of 50 years and an interest rate of 5%, any curvature value above 0.71 satisfies Condition 1. Estimates of this parameter are typically around 0.8 and higher (see, for example, Heckman 1976; Heckman et al. 1998; Kuruscu 2006). The following proposition characterizes the behavior of average wages.

PROPOSITION 1 (Stagnation of Average Wages). *In response to SBTC, for all* $\theta'_H > \theta_H$, *the average wage (alternatively, labor productivity)*

(i) *increases in the long run (that is,* $\bar{w}_{\mathrm{LR}} > \bar{w}_{\mathrm{I}}$);
(ii) *falls in the short run (that is,* $\bar{w}_{\mathrm{SR}} < \bar{w}_{\mathrm{I}}$) *if Condition 1 holds.*

Proof. See the Appendix for omitted proofs and derivations. □

It should be emphasized that, for a *marginal* increase in θ_H, Condition 1 is not only sufficient but also necessary for the average wage to decline in the short run. However, if the increase in the price of human capital were larger then the average wage would decline under a weaker condition, making Condition 1 sufficient but not necessary in general.

The transition path of the average wage after SBTC is easy to characterize. Since the stock of human capital increases monotonically over time, it can be seen (by comparing equations (13) and (14)) that, after the initial decline, the average wage also increases monotonically over time. A useful measure of convergence speed is the time

11. It is easy to see (from equations (8) and (9) and the expression for $C(Q)$) that all three parameters influence both the price and investment effects. This discussion of the impact of α on the cost of investment highlights only the stronger of its two effects (even though it affects both the price and investment effects).

it takes for the wage to return to its level before the shock. For $\beta = \delta = 0.98$, assuming an increase in θ_H of 50% implies that the average wage stays below its initial level for 21 years;[12] if $\beta = \delta = 0.96$, the corresponding duration is eleven years. Overall, these back-of-the-envelope calculations suggest that the average wage can stagnate below its initial level for a decade or perhaps longer and that the convergence to the final steady state is likely to be even slower.

Finally, there would be no stagnation in average wages without the endogenous response of investment to SBTC. Thus, the human capital response is essential for these results.

COROLLARY 1. *If individuals' investment behavior did not respond to SBTC (i.e. if $Q'_{js} = Q_{js}$ for all j and s), then the average wage would immediately jump after SBTC from its initial steady-state value to the final steady-state value.*

3.4. Between-Group Wage Inequality (College Premium)

The wages of college graduates declined throughout the 1970s relative to the wages of less-educated individuals. Starting in the early 1980s this trend reversed course, and the college premium increased sharply in the subsequent two decades. For example, the average male college graduate earned 52% more than a high-school graduate in 1970; this premium fell to 41% in 1979 but increased back to 84% by 2000 (Autor et al. 2008). In this section, we characterize the behavior of the college premium in the model and we show that, under Condition 1, SBTC leads to a nonmonotonic change in the college premium that is similar to the change observed in the data.

Consistently with the standard interpretation of the Ben-Porath model, the perspective adopted in this paper is that educational labels merely represent threshold levels for the human capital investment completed. Thus, a *college graduate* is defined as an individual who has invested above a certain threshold, i^*, in a specified number of periods.[13] Since there is a one-to-one relationship between investment time and ability at every age, there is a corresponding threshold ability level above which all individuals $\left(A_j > A_{j^*}\right)$ become college graduates. However, this threshold depends on the price of human capital and will therefore change in response to SBTC. In the analysis, we abstract from these changes in A_{j^*}. This is because allowing for changes in A_{j^*} would affect the ability composition of each education group over time, thereby possibly confounding the effects of changes in the premium on education by changes in the returns to ability. Hence, we fix the ability distribution of each group and analyze how their wages change in response to SBTC.

Let $\bar{Q}_c$ and $E_c[A]$ denote (respectively) the average investment and average ability of college graduates; we define $\bar{Q}_n$ and $E_n[A]$ in an analogous fashion for individuals

12. The formula for the convergence speed used in this calculation is derived in the supplemental (online) Appendix.

13. More formally, the condition can be stated as $\sum_{s=1}^{\tilde{s}} 1\{i_{js} > i^*\} \geq S_c$, where i^* is the investment time threshold (which is equal to 1 in the standard Ben-Porath model), $\tilde{s}$ is the individual's current age, and S_c is the number of years of schooling required to qualify as a college graduate.

without a college degree. From the previous discussion, it is clear that $E_c[A] > E_n[A]$, which by equation (8) also implies $\bar{Q}_c > \bar{Q}_n$. Finally, let $\bar{w}_c$ (respectively, $\bar{w}_n$) be the average wage of college (high-school) graduates. Then, the college premium before SBTC is

$$\omega_{\mathrm{I}}^{*} \equiv \left.\frac{\bar{w}_c}{\bar{w}_n}\right|_{t<t^*} = \frac{\theta_L l + \theta_H H\left(\bar{Q}_c\right) - C(\bar{Q}_c)}{\theta_L l + \theta_H H\left(\bar{Q}_n\right) - C(\bar{Q}_n)}.$$

The college premium in the *short run* (i.e. immediately after SBTC) is given by

$$\omega_{\mathrm{SR}}^{*} \equiv \left.\frac{\bar{w}_c}{\bar{w}_n}\right|_{t=t^*+\varepsilon} = \frac{\left[\theta_L l + \theta_H' H\left(\bar{Q}_c\right)\right] - C(\bar{Q}_c')}{\left[\theta_L l + \theta_H' H\left(\bar{Q}_n\right)\right] - C(\bar{Q}_n')}. \tag{17}$$

In the long run, the premium is given by

$$\omega_{\mathrm{LR}}^{*} \equiv \left.\frac{\bar{w}_c}{\bar{w}_n}\right|_{t\to\infty} = \frac{\left[\theta_L l + \theta_H' H\left(\bar{Q}_c'\right)\right] - C(\bar{Q}_c')}{\left[\theta_L l + \theta_H' H\left(\bar{Q}_n'\right)\right] - C(\bar{Q}_n')}. \tag{18}$$

The following proposition characterizes the behavior of the college premium.

PROPOSITION 2 (Behavior of College Premium). *In response to SBTC, for all $\theta_H' > \theta_H$, the college premium*

(i) *rises in the long run (that is, $\omega_{\mathrm{LR}}^{*} > \omega_{\mathrm{I}}^{*}$);*
(ii) *falls in the short run (that is, $\omega_{\mathrm{SR}}^{*} < \omega_{\mathrm{I}}^{*}$), if Condition 1 holds.*

Despite the similarities between the statements of Propositions 1 and 2, there is an important difference between them. Whereas the stagnation of average wages requires only the endogenous response of human capital investment to SBTC (i.e. that $C(Q)$ increase after the shock), the fall in the college premium requires, in addition, that this response be different across education groups. In other words, if heterogeneity in ability were eliminated from the model, then average wages would still stagnate after SBTC but the college premium would not fall in the short run.[14]

Since college graduates accumulate skills faster than high-school graduates, the college premium increases monotonically toward the new steady-state value after the initial fall. Moreover, it can be easily shown that the response of the college premium is proportional to the ability differential between college and high-school graduates.

COROLLARY 2. *In response to SBTC, the decline (respectively, increase) in the college premium in the short run (long run) is larger when the ability differential between college graduates and high school graduates, $E_c[A]/E_n[A]$, is larger.*

14. It should be clear from this discussion that if individuals did not respond to SBTC then the college premium would immediately jump from its initial to its final steady-state value.

REMARK 1. It is useful to discuss how these theoretical results contrast with some of the quantitative findings in Heckman et al. (1998). In their model, the college premium peaks in the short run and then falls back in the long run. (For example, in Figure 6 of their paper, the log college premium exceeds 55 log points 25 years after SBTC takes effect but falls back to 46 log points in the long-run.) In contrast, Proposition 2 shows that in our model the college premium is *highest in the long run*, a substantively different conclusion from that of their paper. Furthermore, as we show in Section 4, the college premium falls persistently in our model and is lower ten years after the shock than five years after, whereas in their baseline model, the premium *rises* monotonically after a one year fall in their baseline model (at least for the 30 year period plotted in their figure).

Decomposing the College Premium. To understand better the behavior of the college premium, an intuitive discussion is helpful. For the sake of this discussion, assume that there are no differences in ability *within* each education group and that the investment levels are denoted by Q_c and Q_n for college and noncollege groups, respectively. Of course, investment *time* will vary within each education group because of differences in age and hence in potential earnings (see Lemma 1(ii)). Using the expression for investment time in (7), we can rewrite the college premium as

$$\omega^* = \frac{\theta_L l + \theta_H H\left(Q_c\right)}{\theta_L l + \theta_H H\left(Q_n\right)} \times \frac{1-\bar{i}_c}{1-\bar{i}_n} = \underbrace{\frac{l + \left(\theta_H/\theta_L\right) H\left(Q_c\right)}{l + \left(\theta_H/\theta_L\right) H\left(Q_n\right)}}_{G_1} \times \underbrace{\frac{1-\bar{i}_c}{1-\bar{i}_n}}_{G_2}, \tag{19}$$

where all variables are defined as before, but averages are now taken with respect to the group indicated by the subscript.[15]

The first term in the decomposition, G_1, captures the price and quantity effects of changes in θ_H. Both of these effects are larger for college graduates because they have a larger human capital stock and, moreover, their human capital stock increases more after SBTC (though the latter happens only gradually). The key point is that there is no reason for G_1 to behave in any way other than to increase monotonically after SBTC. If there were no investment response in the model, then G_2 would be constant over time and the college premium would be proportional to G_1 and would also increase monotonically.

The *differential* investment response captured by G_2 is thus crucial for the initial decline in the college premium. There are two reasons for the initial decline in G_2. First, after SBTC, college graduates increase their investment time more than high-school graduates. In the long run this follows because $d^2 i_{js}/d\theta_H dA_j > 0$, as mentioned

15. The variable $\bar{i}_c$ is the weighted average of the investment time of college graduates, where the weights are given by the ratio of each individual's potential earnings $(\theta_L l + \theta_H Q_c(s-1))$ to the average potential earnings of that group $(\theta_L l + \theta_H(\delta/(1-\delta))\bar{Q}_c)$. The definition of $\bar{i}_n$ is analogous.

before. The same can be shown also for the short run.[16] A second and reinforcing effect is that the initial *level* of investment time is larger for college graduates. As a result, even the same *amount* of increase in investment time would cause a decline in $\left(1-\bar{i}_c\right)/\left(1-\bar{i}_n\right)$. Overall, then, the college premium falls initially because G_2 (which depends on the jump variables, $\bar{i}_c$ and $\bar{i}_n$) falls quickly, but the premium recovers as G_1 gradually increases over time.

College Premium within Age Groups. A well-documented fact is that the behavior of the college premium in the United States during this period has been different for different experience groups (Murphy and Welch 1992). In particular, these authors show that both the fall and rise in the overall college premium were largely attributable to individuals with less experience, as the fall and rise in the premium among more experienced individuals were much less dramatic. Similarly, Card and Lemieux (2001) focus on age groups (rather than experience) while examining data from the United Kingdom and Canada in addition to the United States. They find that the same pattern emerges in these countries as well.

To examine this issue, we now look at the college premium among s-year-old individuals. This is given in the initial steady state by

$$\omega_{\mathrm{I}}^{*}(s)=\frac{\theta_L l+\theta_H \bar{Q}_c(s-1)-C(\bar{Q}_c)}{\theta_L l+\theta_H \bar{Q}_n(s-1)-C(\bar{Q}_n)}. \tag{20}$$

Similarly, the premium in the short run and in the long run after SBTC is defined analogously to equations (17) and (18).

PROPOSITION 3 (Behavior of College Premium within Age Groups). *Define*

$$\underline{s}=1+\frac{\alpha\delta\beta}{1-\delta\beta} \quad \text{and} \quad \bar{s}=1+\frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}.$$

Then, in response to SBTC, the college premium among s-year-old individuals:

(i) *falls in the short run,* $\omega_{\mathrm{SR}}^{*}(s)<\omega_{\mathrm{I}}^{*}(s)$, *if and only if* $s<\bar{s}$;
(ii) *rises in the long run,* $\omega_{\mathrm{LR}}^{*}(s)>\omega_{\mathrm{I}}^{*}(s)$, *if and only if* $s>\underline{s}$.

An important difference between this proposition and the previous one is that here the decline in the college premium for young individuals does not require Condition 1, so it holds under more general conditions than does Proposition 2. Furthermore, from Proposition 3 it is easy to conjecture that whether the *average* college premium falls in the short run will depend on whether there are sufficiently many young individuals

16. More formally, we evaluate how the increase in investment time changes with A: we calculate $d^2 i/dA d\theta_H'$, which equals a positive constant times

$$\theta_L l+(2\alpha-1)\frac{\delta}{1-\delta}\theta_H'\bar{Q}.$$

It is clear that $\alpha>0.5$ is a sufficient condition for this cross-partial to be positive. When $\alpha\leq 0.5$, this will still hold true if $\theta_L l/\left((\delta/(1-\delta))\theta_H'\bar{Q}\right)$ is sufficiently large.

TABLE 1. Evolution of college premium within age groups after SBTC.

If s satisfies:	College premium within s-year-old individuals: $s \leq \underline{s}$	$\underline{s} < s < \bar{s}$	$\bar{s} \leq s$
Short run: $\log(\omega^*_{\mathrm{SR}}(s)/\omega^*_{\mathrm{I}}(s))$	Declines (<0)	Declines (<0)	Increases (>0)
Long run: $\log(\omega^*_{\mathrm{LR}}(s)/\omega^*_{\mathrm{I}}(s))$	Declines (<0)	Increases (>0)	Increases (>0)

Notes: See Proposition 3 for the definitions of $\underline{s}$ and $\bar{s}$. The table displays the behavior of the college premium for a *marginal* increase in θ_H.

in the population. In fact, this is what condition 1 ensures in Proposition 2: that the average age in the population must be less than $\bar{s}$ (i.e. $(1-\delta)^{-1} < \bar{s}$) is precisely the same as Condition 1.

Proposition 3 partitions the population into three age groups,[17] in each of which the college premium displays a distinct behavior (see Table 1). First, for very young individuals ($s \leq \underline{s}$) the college premium falls both in the short run and in the long run. This makes sense because, after SBTC, a higher fraction of these individuals go to school for a longer duration (get masters, Ph.D. degrees, etc.), thereby lowering the average wage of this group. The second group ($\underline{s} < s < \bar{s}$), who could be viewed as young *workers*, experience a fall in the premium in the short run but a rise in the long run. Finally, older individuals show a very small investment response to SBTC, and owing to the price effect their wages rise both in the short run and in the long run. The last two predictions are consistent with the evidence documented in Card and Lemieux (2001), who show that the bulk of the fall in the college premium in the 1970s took place among younger workers.

To explain the intuition for these results, it is convenient to take a first-order Taylor series approximation to the college premium for s-year-old individuals (in the initial steady state, the short run, and the long run). This yields

$$\log\left(\frac{\omega^*_{\mathrm{SR}}(s)}{\omega^*_{\mathrm{I}}(s)}\right) \approx \underbrace{\left[\left(\theta'_H - \theta_H\right) \times \left(\bar{Q}_c - \bar{Q}_n\right)(s-1)\right]}_{\text{Differential price effect } (>0)} + \underbrace{\left[\left(C\left(\bar{Q}_c\right) - C(\bar{Q}'_c)\right) - \left(C\left(\bar{Q}_n\right) - C(\bar{Q}'_n)\right)\right]}_{\text{Differential investment effect } (<0)}$$

and

$$\log\left(\frac{\omega^*_{\mathrm{LR}}(s)}{\omega^*_{\mathrm{SR}}(s)}\right) \approx \underbrace{\theta'_H \times \left[\left(\bar{Q}'_c - \bar{Q}_c\right) - \left(\bar{Q}'_n - \bar{Q}_n\right)\right](s-1)}_{\text{Differential quantity effect } (>0)}$$

17. The necessity part of the proposition (the "only if" part) applies for a marginal increase in θ_H. For larger increases, $s < \bar{s}$ is a sufficient but not a necessary condition for the college premium to decline. These comments apply to both part (i) and part (ii) of the proposition. The reason we consider this stronger form of Proposition 3 is because doing so allows us to divide the age range into three distinct groups in Table 1.

(up to a constant scaling factor).[18] Adding these two equations shows that the total change in log education premium, $\log\left(\omega^*_{\mathrm{LR}}(s)/\omega^*_{\mathrm{I}}(s)\right)$, is simply given by the sum of the price, investment, and quantity effects. Comparing these last two expressions with their counterparts derived previously for the average wage (equations (15) and (16)), we note that the only change here is the appearance of *double differences*. For example, the price effect here results from the *differential* impact of the increase in θ_H on the human capital stocks of college and noncollege workers; and the same goes for investment and quantity effects. However, inspecting these two approximations also shows that the three effects have the same signs on the college premium as they had on the average wage. This is because college-educated workers have higher ability on average and therefore (i) have a larger human capital stock before the shock (resulting in a positive price effect), (ii) increase their investment by more after the shock (negative investment effect), and (iii) in the long run, experience a larger increase in their human capital stock (positive quantity effect).

To understand the behavior of the college premium among different age groups, two points should be noted. First, the price and quantity effects on the college premium increase with age—observe the multiplicative $(s-1)$ terms that appear in these two effects—whereas the investment effect does not vary with age. In the short run, then, the constant investment effect dominates the price effect for younger individuals but not for older ones, who experience a larger price effect ($s \geq \bar{s}$). The formal proof in the Appendix establishes that $\bar{s} > 1$, so the college premium does fall among a group of young individuals in the short run but not for the old. Second, as before, the only difference in the long run is the additional quantity effect. As a result, some of the relatively younger individuals ($\underline{s} < s < \bar{s}$) also experience a rise in the premium, and only the youngest see a decline in the long run.

3.5. Within-Group Wage Inequality

A well-known empirical fact in the US data is that wages in the higher percentiles in 1963 experienced high growth in the subsequent four decades whereas the opposite happened at lower percentiles (Juhn et al. 1993, Figure 3; Autor et al. 2008). Consequently, the rise in wage inequality occurred via a nearly uniform stretching out of the entire wage distribution. The following proposition states that this same outcome is predicted by our model.

PROPOSITION 4 (Within-Group Inequality). *Let $w_{\mathrm{I}}(\Omega)$ and $w_{\mathrm{LR}}(\Omega)$ denote, respectively, the wage at the Ωth percentile of the wage distribution before SBTC and in the new steady state after SBTC. Then, $w_{\mathrm{LR}}(\Omega)/w_{\mathrm{I}}(\Omega)$ is increasing in Ω.*

Juhn et al. (1993, Figure 5) show that the same fanning out of the wage distribution is obtained when one conditions on a particular age group. This is true also in our model.

18. To the extent that the change in the college premium is large in response to SBTC, these approximations would not be accurate enough for quantitative analysis. However, they are useful for explaining the intuition of the results, for which purpose they are employed here. However, we do not use them in any formal proof or derivation.

The intuition is simple and can be seen from Figure 1, which shows that a higher price of human capital stretches out the wage distribution at every age (above a threshold) without a change in the relative ranking of individuals. Therefore, individuals (except for the very young) who earn high wages before SBTC also experience a larger increase in their wages after SBTC. The next corollary states this result. (The proof is similar to that of Proposition 4 and is therefore omitted.) Let $w_{\mathrm{I}}(\Omega|s)$ and $w_{\mathrm{LR}}(\Omega|s)$ be the analogs of $w_{\mathrm{I}}(\Omega)$ and $w_{\mathrm{LR}}(\Omega)$ that are conditional on age.

COROLLARY 3. *$w_{\mathrm{LR}}(\Omega|s)/w_{\mathrm{I}}(\Omega|s)$ is increasing in Ω when $s > \underline{s}$.*

3.6. Overall Wage Inequality

As mentioned already, the movement in opposite directions of overall and between-group inequality in the 1970s has received much attention in the literature. In this section we analyze the behavior of overall wage inequality in response to SBTC both in the short run and in the long run. We show that, under a simple sufficient condition, overall inequality *rises* in the short run. Together with the fact that the college premium *falls* in the short run (under Condition 1), this shows that our model is consistent with the joint behavior of college premium and overall wage inequality. To show this, we first remark that the cross-sectional variance of wages can be written as

$$\mathrm{Var}(w) = \left(n_1 \mathrm{Var}(A) + n_2 E\left[A\right]^2\right) \times \theta_h^{2/(1-\alpha)}, \tag{21}$$

where the coefficients' n_i are all positive in the rest of the text (the exact expressions are provided in the supplemental Appendix). Then the coefficient of variation of wages, which we denote by $\mathrm{CV}(w)$, can be written as

$$\mathrm{CV}(w) \equiv \frac{\mathrm{Std}(w)}{\bar{w}} = \left(n_1 \mathrm{Var}(A) + n_2 E\left[A\right]^2\right)^{1/2} \times \left(\frac{\theta_L l}{\theta_H^{1/(1-\alpha)}} + n_4 E\left[A\right]\right)^{-1}, \tag{22}$$

where "Std" denotes the cross-sectional standard deviation. This expression shows that wage inequality is driven by two sources. First, heterogeneity in learning ability (captured by $\mathrm{Var}(A)$) creates wage differences within every age group and increases wage inequality. Second, a higher average learning ability ($E[A]$) generates more wage growth over the life cycle (and hence larger wage differences across age groups) thereby increasing the variance in the numerator; but it also increases the average wage, thereby increasing the denominator. This means that, although the effect of ability heterogeneity on inequality is always positive, the effect of average ability is ambiguous.

Expression (22) shows that the coefficient of variation is increasing in θ_H. This implies that, compared with the initial steady state, wage inequality will be higher in the new steady state after SBTC. In fact, this result could be anticipated from Figure 1, which shows the widening of the wage distribution within age groups as well as the steepening of profiles across age groups when θ_H is higher.

Finally, it is also possible to derive a (complicated) expression for the coefficient of variation of wages immediately after SBTC and so prove the following result. (See supplemental Appendix for the proof.)

PROPOSITION 5 (Rise in Overall Wage Inequality). *In response to SBTC, for all $\theta'_H > \theta_H$, wage inequality (as measured by the coefficient of variation):*

(i) *increases in the long run (that is,* $\mathrm{CV}_{\mathrm{LR}}(w) > \mathrm{CV}_{\mathrm{I}}(w)$*);*
(ii) *increases in the short run (that is,* $\mathrm{CV}_{\mathrm{SR}}(w) > \mathrm{CV}_{\mathrm{I}}(w)$*) if* $\beta = 1$.

Although $\beta = 1$ is sufficient for an increase in wage inequality in the short run, it is far from necessary. When $\beta < 1$ there remains a wide range of parameters that lead to an increase in wage inequality in the short run, but we have not been able to find a simple sufficient condition in that case.

When taken together, Propositions 2 and 5 show that overall wage inequality and between-group inequality (college premium) move in opposite directions in the short run after SBTC. As noted earlier, this result is consistent with evidence from the US data, and the present framework delivers this outcome despite a single driving force.[19]

3.7. Comparison with the Standard Ben-Porath Model

To understand the role of the two-factor structure (with raw labor and human capital) proposed in this paper, it is instructive to examine the response to skill-biased technical change under the standard, one-factor Ben-Porath model. The following proposition characterizes this case.

PROPOSITION 6. *Consider the standard Ben-Porath model with heterogeneity in ability and initial human capital. Individuals solve*

$$\max_{\{i_{js}\}_{s=1}^{S}} \left[\sum_{s=1}^{S} \left(\frac{1}{1+r} \right)^{s-1} \theta_H h_{js}(1 - i_{js}) \right] \tag{23}$$
$$\text{subj. to}: Q_{js} = \tilde{A}_j (h_{js} i_{js})^{\alpha} \quad \text{with } h_{j,0} > 0.$$

19. An important reason for the rise in overall inequality—despite a falling college premium—is that, in the short run, average wage profiles shift downward and become *steeper*, increasing the wage differences between the young and the old. To see this, consider the average wage at age s, immediately after SBTC: $\bar{w}_{\mathrm{SR}}(s) = [\theta_L l + \theta'_H(\bar{Q}(s-1))] - C(\bar{Q}')$; here $\bar{Q}(s-1)$, the *stock* of human capital, is relatively fixed in the short run. We already mentioned that the price effect is larger for older individuals who have a higher human capital stock compared to younger individuals. This differential price effect steepens the wage profiles immediately after SBTC. Second, the investment effect $C(\bar{Q}')$ is independent of age, so it reduces the level of wages for all age groups by the same amount; this shifts the experience profile of wages downward and thus increases *percentage* differences between the young and the old, which further increases inequality across different age groups. As a result, overall wage inequality will increase because of a steepening age profile of wages. The steepening of wage profiles after SBTC is consistent with the US data: Katz and Murphy (1992, Table 1) report that, between 1971 and 1987, the wages of workers with 1–5 years of experience fell by 10.2% more than the wages of workers with 26–35 years of experience.

Then, for all $\theta'_H > \theta_H$, the following statements hold.

(i) *The average wage does not change:* $\bar{w}_{\mathrm{I}} = \bar{w}_{\mathrm{SR}} = \bar{w}_{\mathrm{LR}}$.
(ii) *The college premium does not change:* $\omega^*_{\mathrm{I}} = \omega^*_{\mathrm{SR}} = \omega^*_{\mathrm{LR}}$.
(iii) *The college premium conditional on age does not change:* $\omega^*_{\mathrm{I}}(s) = \omega^*_{\mathrm{SR}}(s) = \omega^*_{\mathrm{LR}}(s)$ *for all s.*
(iv) *Within-group inequality in the long run does not change:* $w_{\mathrm{LR}}(\Omega)/w_{\mathrm{I}}(\Omega)$ *is independent of* Ω.
(v) *Overall wage inequality does not change:* $\mathrm{CV}_{\mathrm{I}}(w) = \mathrm{CV}_{\mathrm{SR}}(w) = \mathrm{CV}_{\mathrm{LR}}(w)$.

This proposition shows that none of the results that we derived for wage inequality holds true in the standard Ben-Porath model *even* allowing for heterogeneity (of arbitrary form) in ability and initial human capital. The intuition for this result can be understood simply by comparing the optimality condition obtained in the standard Ben-Porath framework to our baseline model. Specifically, the first-order condition for (23) is

$$\theta_H(t)C'_j(Q_{js}) = \left\{\frac{\theta_H(t+1)}{1+r} + \frac{\theta_H(t+2)}{(1+r)^2} + \cdots + \frac{\theta_H(t+S-s-1)}{(1+r)^{S-s-1}}\right\}. \quad (24)$$

Now suppose that the economy is in steady state with $\theta_H(t) = \bar{\theta}_H$ for all t, and consider the effect of a surprise, one-time but permanent increase in the wage rate. This change will have *no effect* on investment behavior because a permanently higher θ_H will increase both the cost and the benefit of investment (i.e. the left- and right-hand sides of (24)) by exactly the same amount. Therefore, the *price* of human capital in the standard Ben-Porath model does not capture what we think of as a *return* on human capital investment. In contrast, a rise in θ_H (e.g. due to SBTC) in our framework increases the benefit of human capital investment (the right-hand side of (5)) relative to the cost of investment and therefore increases the incentives to invest in human capital permanently.

3.8. Wage Inequality and Consumption Inequality

The measures of wage inequality discussed so far (i.e. overall, between-group, and within-group) are based on the distribution of wages at one moment in time. In that sense, they provide *snapshot* measures of inequality. For many purposes, however, it is of interest to know whether the observed changes in these snapshots imply a parallel change in *lifetime* income inequality. A surprising empirical finding is that the rise in consumption inequality—which can be thought of as a proxy for lifetime income inequality—has been muted compared with the rise in wage inequality during this period (Attanasio et al. 2004; Krueger and Perri 2006). Moreover, the change between the 90th and 50th percentiles of the consumption distribution has not tracked the large rise in the 90–50 percentile wage differential. Autor et al. (2004) document this fact and find it puzzling.

We now examine the behavior of lifetime wage income inequality in response to SBTC. Note that, under the assumptions made so far, individuals choose a constant consumption path over their life cycle. Hence, consumption inequality equals lifetime wage inequality, and we use the two interchangeably. It can be shown (see the supplemental Appendix) that the variance of consumption equals $\mathrm{Var}(c) = n_3 \,\mathrm{Var}(A)\theta_h^{2/(1-\alpha)}$. This expression differs from that for the variance of wages (equation (21)) in two ways. First, as noted earlier, part of the variance of wages is due to the differences in wages across age groups (when $E[A] > 0$). This effect is not present in the variance of consumption because individuals with the same ability will consume the same amount regardless of their age (since they have the same lifetime income). Therefore, the variance of consumption is driven by heterogeneity in learning ability, which is the only source of permanent differences in lifetime incomes. Second, a given heterogeneity in learning ability ($\mathrm{Var}(A)$) results in less consumption inequality than wage inequality (that is, $n_3 < n_1$). The is because individuals with high wages later in life are precisely those who made larger investments and accepted lower wages early on. As a result, consumption inequality is lower than wage inequality.

We now use these results to examine how wage and consumption inequality *change* relative to each other in response to SBTC. In particular, when the subjective time discount rate is zero, we can show that $\mathrm{CV}(w)^2$ will always increase more than $\mathrm{CV}(c)^2$ in response to SBTC regardless of other parameter values. The next proposition formalizes this result.

PROPOSITION 7 (Rise in Wage and Consumption Inequality). *Assume that* $\beta = 1$. *In response to SBTC, for all* $\theta'_H > \theta_H$, *wage inequality rises more than consumption inequality in the long run; that is,* $\mathrm{CV}_{\mathrm{LR}}(w)^2 - \mathrm{CV}_{\mathrm{I}}(w)^2 > \mathrm{CV}_{\mathrm{LR}}(c)^2 - \mathrm{CV}_{\mathrm{I}}(c)^2$.

To prove this proposition, note that when $\beta = 1$ first we have $\bar{c} = \bar{w}$ (by equations (10) and (11)). Then combining $\bar{c} = \bar{w}$ with (10) and (21), we obtain

$$\mathrm{CV}(w)^2 - \mathrm{CV}(c)^2 = n_2 \left(\mathrm{Var}(A) + E\,[A]^2\right) \left(\frac{\theta_L l}{\theta_H^{1/(1-\alpha)}} + n_4 E\,[A]\right)^{-2},$$

for a given steady state. This expression is increasing in θ_H. Therefore, it is higher in steady state after SBTC than in the initial steady state, which completes the proof.

Observe that the difference between wage and consumption inequality increases more with an increase in θ_H when $\mathrm{Var}(A)$ is larger. Furthermore, if SBTC is modeled as involving a simultaneous fall in θ_L, then the difference between wage and consumption inequality would increase even further after SBTC. Although we have not been able to extend this result to the more general case with $\beta < 1$, in simulations we have found wage inequality to increase (substantially) more than consumption inequality for a wide range of parameter values.

3.9. College Enrollment versus On-the-Job Training

Although our main focus in this paper is on evolution of the wage distribution, the model also makes predictions about the behavior of college enrollment—in particular, the model predicts that such enrollment will increase in response to SBTC. To show this, we first define an individual to be currently *enrolled in college* if his investment time exceeds a threshold level i^* (as with the standard interpretation of the Ben-Porath model). Now, let Π_m for $m = I, SR$, and LR denote the fraction of population enrolled in college in the initial steady state, the short run and the long run respectively. We have the following result. (See the supplemental Appendix for the proof.)

LEMMA 2. (College Enrollment). $\Pi_{\mathrm{SR}} > \Pi_{\mathrm{LR}} > \Pi_{\mathrm{I}}$ *for all s. Thus, after SBTC, the college enrollment rate increases in the long run but increases even more in the short run.*

Enrollment is highest in the short run because the opportunity cost of investing—which is determined by current potential earnings—does not change immediately after SBTC even as the potential future benefits (determined by θ'_H) are increasing. Over time, as the price of human capital rises, investment becomes more costly and college enrollment falls to its final steady-state level, which is still higher than the initial level.

Although overall college enrollment increased significantly in the United States from 1970 to 2000 (especially when female college enrollment is included), which is consistent with Lemma 2's prediction for the long-run trend, the enrollment rate was actually stagnant in the 1970s, which is at odds with the model's prediction for the short run (see Card and Lemieux 2001). Yet this counterfactual implication follows from our assumption that SBTC happens in a completely *disembodied* fashion: the productivity of all human capital rises at the same rate regardless of when it is acquired. As a result, the incentive to invest is strongest immediately after SBTC begins and strongest among young individuals; the result is increased college enrollment. Although our assumption of disembodied SBTC proved to be analytically convenient, in reality some types of technical change *are* embodied in new types of human capital. With embodied SBTC, however, it is easy to show that college enrollment does not necessarily rise and, in fact, may fall in the short run. At the same time, on-the-job investment still rises; this causes the college premium to fall in the short run (and rise in the long run), as in the baseline case analyzed earlier. In the working paper version of this paper (Guvenen and Kuruscu 2007, Section 4), we examine the model just described with embodied SBTC and prove these results.[20] Thus, if part of SBTC takes place in an embodied fashion, then the counterfactual implications about college enrollment in the short run could be overturned while retaining the plausible implications of the model for the evolution of the wage distribution.

4. A Quantitative Exploration

The theoretical results presented in Section 3 showed that the wage distribution trends observed in the US data are also robust qualitative features of our baseline model. An important question is whether the model is also *quantitatively* consistent with the empirical magnitudes documented in the literature. Clearly, such an analysis

20. The drawback of this alternative formulation (with embodied SBTC) is that it is not nearly as analytically tractable as our baseline framework. We thus do not pursue this extension further here.

requires relaxing several stark assumptions made here for analytical tractability. In a companion paper (Guvenen and Kuruscu 2010), we carry out a detailed quantitative assessment of this model in which we relax the *perfect foresight* assumption (and, instead, consider Bayesian learning with several different prior beliefs) while allowing for aggregate uncertainty in skill prices, heterogeneity in initial raw labor endowments, a nonconvex choice set for investment time (to better distinguish schooling from on-the-job investment), and so on. These additional features allow for a more realistic calibration, but it is useful to understand how much mileage we can get with the stylized model studied in this paper.

To help answer this question, in this section we simulate the response of the basic framework laid out in Section 2 to skill-biased technical change. Recall that the framework has a finite planning horizon, which allows us to illustrate whether the perpetual youth assumption made in Section 3 for tractability purposes has any substantive effects on our conclusions. We assume that SBTC happens as a one-time change in the *growth* rate (instead of the *level*) of the skill price, which is more consistent with how SBTC has been modeled in the empirical literature. In addition, we focus on some important statistics that are not studied in Guvenen and Kuruscu (2010) and also provide a direct comparison to Heckman et al. (1998).[21] Furthermore, we compare the model to the wage data on *all* workers—unlike our companion paper, which focuses on male workers only. This provides a more stringent test for the model since it checks whether the human capital mechanisms studied in this paper play an important role in female wage trends.

As for the calibration, we choose the three free parameters—the mean and standard deviation of a uniform ability distribution and the maximum investment allowed on the job—to match the 1965–1969 averages of (i) the variance of log wages net of idiosyncratic shocks (0.10), (ii) the college premium (0.38), and (iii) the relative supply of college graduates (24% of the population).[22] Finally, the cumulative change in SBTC (θ_H/θ_L) from 1970 to 2000 is set to 12% in order to match the 12.5 log points rise in the variance of log wages in the US data during the same period. No other empirical moment after 1970 is targeted.[23]

Table 2 reports the evolution over time of the upper (log 90–50) and lower (log 50–10) tail inequality from 1970 to 2000. In all columns, we normalize the empirical statistics and the simulated counterparts to 0 in 1970 to facilitate comparison of the

21. The college premium is the only statistic that is studied both in this section and in our companion paper, so we include it here for completeness of discussion. Of course, the model used in this paper (and its calibration) is different from that used in our companion paper.

22. The calibrated values of the mean and standard deviation of the ability distribution are 0.105 and 0.249, respectively. Also, i^* is set to 0.8 and any investment level above this for two years or more qualifies an individual as a college graduate. With this definition, the fraction of college graduates changes (increases, due to SBTC) over time—unlike what is assumed in Section 3. The level of raw labor is a scaling parameter. See Guvenen and Kuruscu (2010) for further discussion on the choices of these empirical targets.

23. The data used to compute the empirical statistics are obtained from Lemieux (2006) (available from the *American Economic Review*'s website) and Autor et al. (2008) (available from David Autor's website). Because, in some cases we process these data further, we provide the final form of the data sets that used in Section 4 as supplementary material to this paper (available on this journal's website).

TABLE 2. Change in overall and between-group inequality: 1970–2000.

	Overall inequality × 100				Log education premium × 100	
	Log 90–50		Log 50–10			
Year	Data	Model	Data	Model	Data	Model
1970	0.0	0.0	0.0	0.0	0.0	0.0
1975	1.1	3.8	1.6	2.3	−2.9	−1.6
1980	3.1	9.1	0.3	−0.1	−6.0	−1.9
1985	7.1	16.3	6.0	2.0	2.3	1.6
1990	9.0	25.1	10.2	4.6	8.4	9.0
1995	15.2	33.2	8.7	7.4	12.8	14.7
2000	18.7	36.9	9.7	9.2	18.1	18.0

Notes: For a given year t shown in column 1, the reported statistics are calculated by averaging the values for year $t-1$, t, and $t+1$ to smooth out noise in the data. All statistics have been normalized to 0.0 in 1970, so the figures for year 2000 also represent the cumulative change from 1970 to 2000.

model and data trends. A well-documented observation—also evident here—is that, over the entire period, the rise in wage inequality has been more pronounced at the upper end: twice the total rise at the lower end. The model reproduces this behavior, with the rise at the lower end nearly matching the data (9.2 log points versus 9.7 in the data). However, the rise implied by the model at the upper end overstates the rise in the data (36.9 points versus 18.7 points in the data).

Another empirical fact is that the college premium falls in the US data during the 1970s and is lower in 1980 than it is in 1975 or 1970. The same persistent fall is also apparent in the model, although the magnitude is somewhat smaller than in the data.[24] This result is difficult to generate using the standard Ben-Porath model; indeed, Heckman et al. (1998) find that the college premium falls by very little and for only one year after SBTC takes effect. Although they show that an extension of their model (with cohort-size variation over time) is able to generate a more prolonged decline, this result is due to the large increase in the relative supply of college graduates (due to Baby Boom cohorts), which is the same supply–demand mechanism studied in Katz and Murphy (1992). In contrast, this channel is completely (and intentionally) shut down in our model by assuming perfect substitution in the production function. Thus, the differential investment response to SBTC alone is responsible for the decline in the college premium. Although this mechanism is discussed in Heckman et al. (1998), their model does not generate a quantitatively large investment response to induce a large and persistent decline in the college premium.

We next turn to the evolution of within-group inequality over time. Recall that Proposition 4 characterized the change in inequality within each wage percentile only in the long run after SBTC. But as mentioned in Section 1, the opposite behavior of within-group inequality and the college premium in the 1970s (i.e. in the short run) has attracted much attention and has been viewed as puzzling by Juhn et al. (1993)

24. In Guvenen and Kuruscu (2010) we show that uncertainty and Bayesian learning further amplify the fall in the college premium after SBTC.

TABLE 3. Change in within-group inequality: 1970–2000.

	Residual variance (×100)		$\sigma^2(\log(W)) \times 100$ within: College grads		High-school grads	
	Data	Model	Data	Model	Data	Model
1970	0.0	0.0	0.0	0.0	0.0	0.0
1975	0.5	0.6	0.0	10.5	0.0	−0.6
1980	1.9	1.0	2.3	11.1	0.7	−0.5
1985	4.6	1.8	4.0	12.2	3.0	0.6
1990	6.0	2.8	6.3	14.0	4.3	1.8
1995	7.7	3.6	9.3	15.4	4.6	2.8
2000	8.9	3.9	10.4	15.2	4.7	2.7

and the subsequent literature. The quantitative analysis here allows us to address this question directly. Table 3 reports two different measures of within-group inequality. We first regress raw wages on dummies for age and education (high-school versus college graduates). The second and third columns report the variance of the regression residuals in the US data and in simulated data, respectively. We have already mentioned that, despite the fall in the college premium in the 1970s, within-group inequality does not fall and, in fact, rises in the US data for this period. The same monotonic rise throughout the period is generated by our model. Hence, the model is consistent with the contrary trends of within- and between-group inequality in the 1970s despite relying on a single driving force—in contrast to Juhn et al.'s (1993) conjecture mentioned in the Introduction.

We next turn to another measure of within-group inequality: the variance of log wages calculated separately within the samples of college and high-school graduates. As seen in the fourth and sixth columns of the table, in the data this variance rises by twice as much within the former group as in the latter. The model is broadly consistent with this observation: the variance rises by 15.2 log points within college graduates (versus 10.4 in the data) and by 2.7 log points within high-school graduates (versus 4.7 in the data). This last result stands in contrast to Heckman et al. (1998), who find that virtually all the rise in the variance of log wages in their model occurs because of a large rise in the variance within high-school graduates, and the variance within college graduates falls almost monotonically starting one year after SBTC takes effect (see their Figure 13).

The results of these illustrative simulations are quite encouraging, given the extremely stark nature of our model and a host of important features that are omitted. Overall, the model generates a sustained fall, as well as a subsequent sustained rise, in the college premium; it generates a persistent rise in all other measures of wage inequality, and this happens more at the top of the ability distribution (i.e. in the 90–50 differential and among college graduates) than at the bottom. Furthermore, although we do not elaborate on this here, the model's implications for the stagnation of average wages and the small rise in consumption inequality are also in line with the empirical trends observed in the US data during this period (see Guvenen and Kuruscu 2010).

5. Conclusions

This paper has studied the implications of a tractable overlapping-generations model of human capital accumulation for several empirical trends in the evolution of the wage distribution since the early 1970s. The key element in the model is the interaction between skill-biased technical change—which is interpreted broadly as a rise in the price of human capital—and heterogeneity in the ability to accumulate human capital. Because of this heterogeneity, the responses of different individuals to SBTC differ systematically from each other. As a result, the model generates rich behavior in the relative wages of individuals as a function of their age and ability, thereby creating interesting dynamics in the evolution of the wage distribution. The model is consistent with: the joint behavior of the college premium (which fell first and then rose strongly) and overall inequality (which rose throughout this period), despite the model's reliance on a single driving force; the monotonic rise in within-group inequality; the stagnation of average wages for an extended period of time; and the small increase in lifetime (consumption) inequality in spite of the large rise in wage inequality.

A potentially important feature not considered in our paper is incomplete markets. The simplest way to think about this issue (without introducing uncertainty) is to disallow borrowing. Then, it is easy to see that, for a given ability distribution and price of human capital, there will be less cross-sectional wage inequality. This is because individuals will not be able to smooth consumption over time by borrowing and so will be less inclined to invest in human capital, which steepens the life-cycle wage profile. Of course, one can recalibrate the ability dispersion and/or increase the price of human capital and thereby generate greater wage inequality. The same is true for the *change* in wage inequality over time. Thus, it is not clear how critical is the assumption of complete markets/perfect capital markets. The implications for consumption may be more important, however: with incomplete markets, an individual's consumption will be more closely linked to his income; hence SBTC is likely to imply a higher rise in consumption inequality than in the complete markets version. This may not be bad news, since in Guvenen and Kuruscu (2010) we find that the calibrated version of the present model with complete markets understates the (already small) rise in consumption inequality observed in the US data. Therefore, incomplete markets could bring the model closer to the data along this dimension. This possibility is only conjectural for now and the difficult but important task of integrating our framework with an incomplete markets environment is left for future work.

Appendix: Derivations and Proofs of Propositions

Proof of Proposition 1. By substituting the optimal investment level we obtain the initial average wage (before the shock) as

$$\bar{w}_{\mathrm{I}} = \theta_L l + \left(\theta_H \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)}\right) E\left[A\right]$$

and in the short run as

$$\bar{w}_{\mathrm{SR}} = \theta_L l + \left(\frac{\delta}{1-\delta}\theta'_H\left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} - \frac{\alpha\delta\beta}{1-\delta\beta}\theta'_H\left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H\right)^{\alpha/(1-\alpha)}\right)E[A].$$

Then $\bar{w}_{\mathrm{SR}} < \bar{w}_{\mathrm{I}}$ if and only if

$$\frac{\theta'_H}{\theta_H}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\theta'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

To see under what conditions this inequality is satisfied, consider the function

$$f(x) = x\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}x^{\alpha/(1-\alpha)}\right).$$

Notice that a skill-biased technical change is equivalent to increasing $x \equiv \theta'_H/\theta_H$ above 1. Therefore, if $f'(1) < 0$ and $f'(x) < 0$ for $x > 1$, then the inequality previously displayed is satisfied and $w_{\mathrm{SR}} < w_{\mathrm{I}}$. Therefore,

$$f'(1) = \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0$$

and $f'(x) < 0$ for $x > 1$ if and only if

$$\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0.$$

□

Proof of Proposition 2. Let $\varphi \equiv \bar{Q}_c/\bar{Q}_n = \bar{Q}'_c/\bar{Q}'_n = E_c[A]/E_n[A]$. Substitute $\bar{Q}_c = \varphi\bar{Q}_n$ and $C(\bar{Q}_n) = (\alpha\delta\beta/(1-\delta\beta))\theta_H\bar{Q}_n$ into the college premium to obtain

$$\omega^*_{\mathrm{I}} = \frac{\theta_L l + \varphi\theta_H\bar{Q}_n\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}{\theta_L l + \theta_H\bar{Q}_n\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)} \quad \text{and} \quad \omega^*_{\mathrm{LR}} = \frac{\theta_L l + \varphi\theta'_H\bar{Q}'_n\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}{\theta_L l + \theta'_H\bar{Q}'_n\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}.$$

Since $\theta'_H\bar{Q}'_n > \theta_H\bar{Q}_n$, it follows that if the function

$$g(x) = \frac{\theta_L l + \varphi x}{\theta_L l + x}$$

is increasing in x then $\omega^*_{\mathrm{LR}} > \omega^*_{\mathrm{I}}$. We have

$$g'(x) = \frac{\varphi\theta_L l - \theta_L l}{(\theta_L l + x)^2} g'(x) > 0 \quad \Leftrightarrow \quad \varphi > 1.$$

Then $\omega^*_{\mathrm{LR}} > \omega^*_{\mathrm{I}}$ iff $E_c[A] > E_n[A]$. The premium in the short run can be written as

$$\omega^*_{\mathrm{SR}} = \frac{\theta_L l + \varphi\left(\frac{\delta}{1-\delta}\theta'_H\bar{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H\bar{Q}'_n\right)}{\theta_L l + \frac{\delta}{1-\delta}\theta'_H\bar{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H\bar{Q}'_n} = \frac{\theta_L l + \varphi x_{\mathrm{SR}}}{\theta_L l + x_{\mathrm{SR}}},$$

where

$$x_{\mathrm{SR}} = \frac{\delta}{1-\delta}\theta'_H \bar{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H \bar{Q}'_n.$$

Let

$$x_{\mathrm{I}} = \frac{\delta}{1-\delta}\theta_H \bar{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \bar{Q}_n.$$

If $x_{\mathrm{SR}} < x_{\mathrm{I}}$, then $\omega^*_{\mathrm{SR}} < \omega^*_{\mathrm{I}}$. We will therefore characterize the condition under which $x_{\mathrm{SR}} < x_{\mathrm{I}}$. Plugging in the optimal investment choices, we can show that $x_{\mathrm{SR}} < x_{\mathrm{I}}$ iff

$$\frac{\theta'_H}{\theta_H}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\theta'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

This is the same condition as in Proposition 1, so $\omega^*_{\mathrm{SR}} < \omega^*_{\mathrm{I}}$ for all $\theta'_H > \theta_H$ if

$$\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0.$$

□

Proof of Proposition 3. The proof is similar to the proof of Proposition 2. Let φ be defined as in the proof of Proposition 2. The premium in the short run can be written as

$$\omega^*_{\mathrm{SR}}(s) = \frac{\theta_L l + \varphi\left(\theta'_H \bar{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H \bar{Q}'_n\right)}{\theta_L l + \theta'_H \bar{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H \bar{Q}'_n} = \frac{\theta_L l + \varphi x_{\mathrm{SR}}}{\theta_L l + x_{\mathrm{SR}}},$$

where

$$x_{\mathrm{SR}} \equiv \theta'_H \bar{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H \bar{Q}'_n.$$

Let

$$x_{\mathrm{I}} \equiv \theta_H \bar{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \bar{Q}_n.$$

The education premium declines in the short run iff $x_{\mathrm{SR}} < x_{\mathrm{I}}$. Thus we have

$$x_{\mathrm{SR}} < x_{\mathrm{I}} \iff \frac{\theta'_H}{\theta_H}\left(s-1-\frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\theta'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < s-1-\frac{\alpha\delta\beta}{1-\delta\beta}.$$

Define the function

$$f_s(x) \equiv x\left(s-1-\frac{\alpha\delta\beta}{1-\delta\beta}x^{\alpha/(1-\alpha)}\right), \tag{A.1}$$

and observe that a skill-biased technical change is equivalent to increasing the ratio $x \equiv \theta'_H/\theta_H$ above 1. Therefore, if we can establish that $f'_s(1) < 0$ then $\omega^*_{\mathrm{SR}}(s) < \omega^*_{\mathrm{I}}(s)$.

By differentiating f_s using (A.1) and evaluating at 1 shows that

$$f_s'(1) < 0 \iff s < 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}.$$

Consequently, $\omega^*_{\mathrm{SR}}(s) < \omega^*_{\mathrm{I}}(s)$ if

$$s < 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}.$$

The college premium in the long run is given by

$$\omega^*_{\mathrm{LR}}(s) = \frac{\theta_L l + \varphi\left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)\theta'_H \bar{Q}'_n}{\theta_L l + \left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)\theta'_H \bar{Q}'_n}.$$

Since $\theta'_H \bar{Q}'_n > \theta_H \bar{Q}_n$ and $\varphi > 1$, the college premium would increase in the long run if $s - 1 > \alpha\delta\beta/(1-\beta\delta)$. □

Proof of Proposition 4. Recall that $w_{s,j} = \theta_L l + \theta_H Q_j(s-1) - C(Q_j)$. Plugging in the optimal investment, we can write $w_{s,j} = \theta_L l + n_5 \theta_H^{1/(1-\alpha)} y$, where

$$n_5 \equiv \left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} \quad \text{and} \quad y = \left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right) A,$$

where y captures the component of wages that varies with age (s) and ability (A). It is clear that $w_{s,j}$ is strictly increasing in y. Hence a worker's relative position in the wage distribution (i.e., Ω) is positively related to y. The wage of this worker in the initial steady state is given by $w_{\mathrm{I}}(y) = \theta_L l + n_5 \theta_H^{1/(1-\alpha)} y$, and the corresponding wage in the long run is $w_{\mathrm{LR}}(y) = \theta_L l + n_5 \theta_H'^{1/(1-\alpha)} y$. It is then easy to show that $w_{\mathrm{LR}}(y)/w_{\mathrm{I}}(y)$ is increasing in y. □

Proof of Proposition 5. See the supplemental Appendix. □

Proof of Lemma 2. See the supplemental Appendix. □

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Data sets (zip file)

Appendix S2. Further proofs (pdf file)

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