Midterm Solution II
Econ 4698, Modern Economic Growth

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Question 1, 20 PTS

After you graduate from U of M, you start your new job as a consultant at Goldman Sachs, where your boss asks your advice on which growth model to use for understanding the determinants of China's economic growth since the 1980s. Answer the following two questions using three sentences for each.

(a)

She is trying to decide between the Solow model and endogenous growth models. Which one should she use? Explain.

Answer:
Since in the 1980's China was in the "Catching-up" phase, you can use Solow.

(b)

Would your recommendation change if she was, instead, interested in the determinants of Germanys or the United States growth performance since the 1980s? Why or why not?

Answer:
Since Germany and the the United States have already on their BGP you can use endogenous growth model.

Question 2, 25 PTS

In the simplified Romer model, we assumed that the production function for new technology (or ideas) was given by: $\dot{A} = \delta L_A^{\lambda} A^{\phi}$.

(a)

Suppose that ϕ is greater than zero. What does this imply for the relationship between the current technology level and the difficulty of discovering new ideas? What is the name of this effect?

Answer:

- This is called "Standing on Shoulders" effect.
- $\phi > 0$ indicates that the productivity of research increases with the stock of ideas that have already been discovered.
- $\phi > 0$ reflects a positive knowledge spillover in research.

(b)

Is it plausible to λ=1? Why or why not? If the answer is no, what is a more plausible assumption?

Answer:
If we set $\lambda = 1$:

$$\frac{\dot{A}}{A} = \delta L_A.$$

If we double the number of researchers and innovators then we get some replication effect. Some of them might replicate the others work. Also stepping on toes argument (congestion stuff...).

(c)

Is it possible to get long-run TFP growth with zero population growth with the production function for ideas given above? Why or why not? Using the appropri-

ate equation(s) explain your reasoning.

Answer:
If $\phi = 1$ we get:

$$\frac{\dot{A}}{A} = \delta L_A^{\lambda}$$

As long as $L_A^{\lambda} \neq 0$ you get a positive growth.

Question 3, 15 PTS

(a)

What is the definition of a “public good? Give an example of a public good.

Answer:

- “In economics, a public good is a good that is both non-excludable and non-rivalrous in that individuals cannot be effectively excluded from use and where use by one individual does not reduce availability to others.”
- Fresh air, National security.

(b)

Give a brief but clear definition of the “Tragedy of Commons. Give an example of a good that suffers from the tragedy of commons.

Answer:

- The problem of “tragedy of commons” happens when a good is rivalrous but has low degree of excludability.
- Forests, Fish in the rivers.

(c)

Yes. Think about digital music.

Question 4, 25 PTS

Consider the standard version of the AK model that we derived by assuming the production function $Y = AK$.

(a)

Consider a tax policy that affects individuals' savings rate. Is this policy going to have a "level effect" or a "growth rate effect" in this AK model? Derive the relevant equation or equations and justify your answer. Explain.

Answer:
The law of motion for capital takes the following form:

$$\dot{K} = sAK - \delta K.$$

We can write:

$$LogY = LogA + LogK.$$

Taking the derivative with respect to time:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = sA - \delta.$$

Therefore

$$g = sA - \delta.$$

Changing s would change g, therefore, it is a **growth rate effect**.

(b)

What is the empirical evidence on the relationship between tax policies and economic growth rates across different countries? Does this evidence support the kind of relationship predicted by the AK model?

Answer:

Question 5, 25+10 PTS

Consider the following simple version of the Romer model. Here are the specifics (the notation below is exactly the same we have used all along):

i. The production function is $Y = K^{\alpha}\bigg(AL_Y\bigg)^{1-\alpha}$

ii. Capital accumulation is $\dot{K} = s_K Y - \delta K$

iii. Population growth $\frac{\dot{L}}{L} = n$

iv. Creation of new ideas: $\dot{A} = \theta A^{\phi} L_A^{\lambda}$

v. Proportion of labor in research:$\frac{L_A}{L} = s_R$

Suppose also that $\lambda < 1$ **and** $\phi > 0$. **Answer the following questions given this information.**

(a)

Show that this model has a Balanced Growth Path. To answer the question you need to write down the condition for balanced growth path and show that it holds using the model's equations. Show your work.

Answer:

Starting from the production function:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}$$

We need to write it somehow as function of $L = L_Y + L_A$. We know that:

$$L_Y = (1 - S_R)L.$$

Substituting this result in the produsction function:

$$Y = K^{\alpha}L^{1-\alpha}(1 - s_R)^{1-\alpha}A^{1-\alpha}.$$

Now lets divide everything by L and see what happens:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha}A^{1-\alpha}(1 - s_R)^{1-\alpha}.$$

Lets rewrite it:

$$y = k^{\alpha}A^{1-\alpha}(1 - s_R)^{1-\alpha}$$

Using the log trick:

$$Log(y) = \alpha Log(k) + (1-\alpha)Log(A) + (1-\alpha)Log(1-s_R)$$

and taking the derivative with respect time:

$$\frac{\dot{y}}{y} = \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A}.$$

Therefore:

$$g_y = \alpha g_k + (1-\alpha)g_A.$$

Assuming $g_k = g_A \equiv g$ we have :

$$g_y = g.$$

(b)

Solve for the growth rate of TFP (or ideas, A) in this model. Show the steps of your work:

Answer:

$$\dot{A} = \theta A^{\phi} L_A^{\lambda}.$$

From the definition of BGP we know that $g_A = \frac{\dot{A}}{A}$ must be constant. Therefore,

$$\frac{\dot{A}}{A} = g_A = \frac{\theta L_A^{\lambda}}{A^{1-\phi}}$$

Now let's use our *Log* trick:

$$Log(g_A) = Log(\theta) + \lambda Log(L_\lambda) - (1-\phi)Log(A).$$

Since g_A and θ are constant:

$$\lambda\frac{\dot{L_A}}{L_A} - (1-\phi)\frac{\dot{A}}{A} = 0$$

We know $\frac{\dot{L_A}}{L_A} = n$ and $\frac{\dot{A}}{A} = g_A$

$$\lambda n - (1-\phi)g_A = 0$$

Therefore,

$$g_A = \frac{\lambda n}{1-\phi}.$$

(c)

Answer the question and provide a 1 sentence explanation for each part] According to the equation you derived in part (b), what does the model imply for the relationship between:

i) population growth and TFP growth?

g_A is a linear function of n. However if $\phi = 1$ then

$$\frac{\dot{A}}{A} = \theta L_A^{\lambda}.$$

ii) The fraction of labor devoted to R&D and TFP growth?

There is no relationship. However, if $\phi = 1$ we get:

$$\frac{\dot{A}}{A} = \theta L_A^{\lambda} = \theta(s_R L)^{\lambda}.$$

Therefore, only in the case of $\phi = 1$ we get a positive relationship between g_A and s_R.

iii) Savings rate and TFP growth

There is no relationship. Changing the saving rate would create a level effect.