Session I
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Econ 4698, Modern Economic Growth

The Law of Motion for Capital

$$\dot{K} = sY - \delta K$$

$$\dot{K} = sK^{\alpha} L^{1-\alpha} - \delta K$$

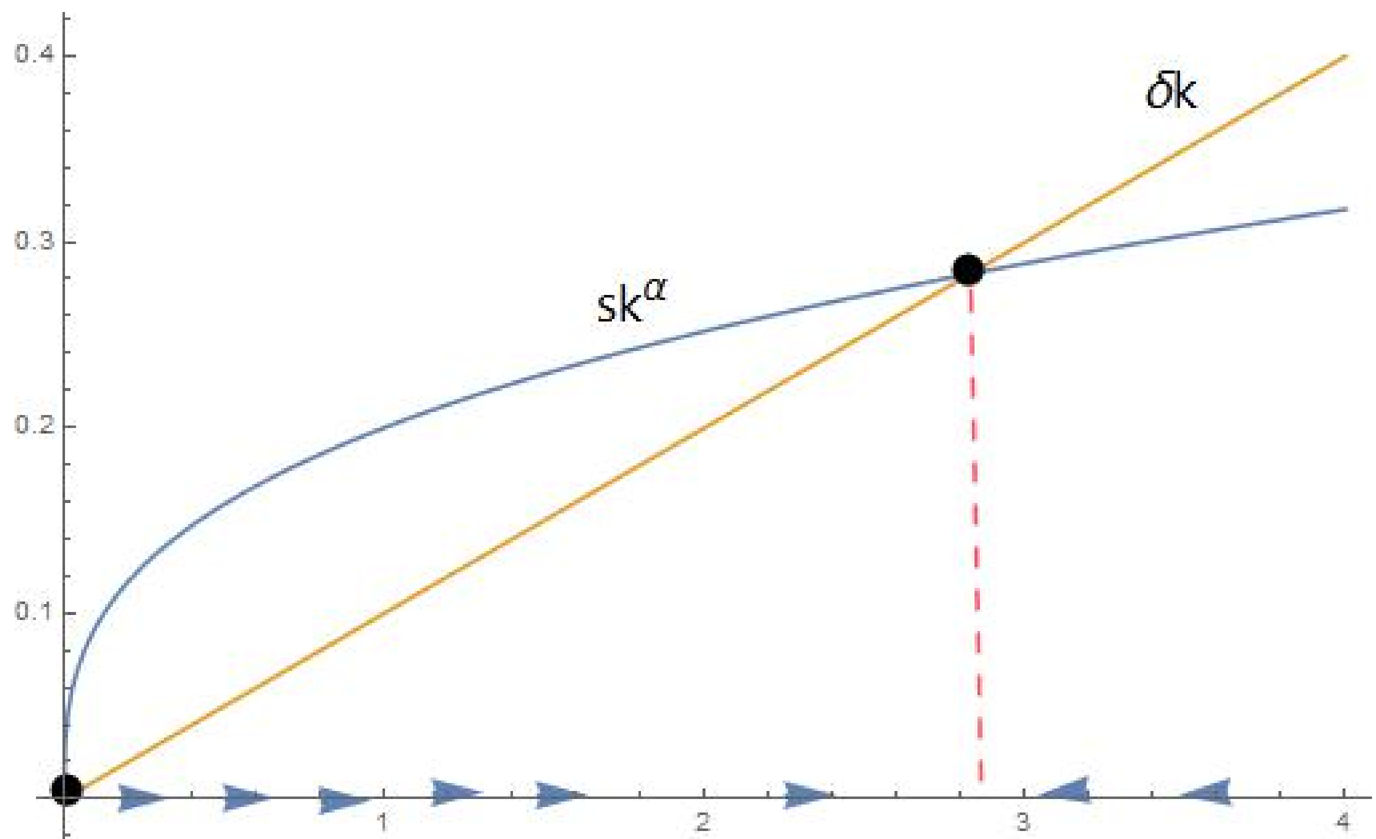
If there is no population growth, we can normalize L

$$L = 1$$

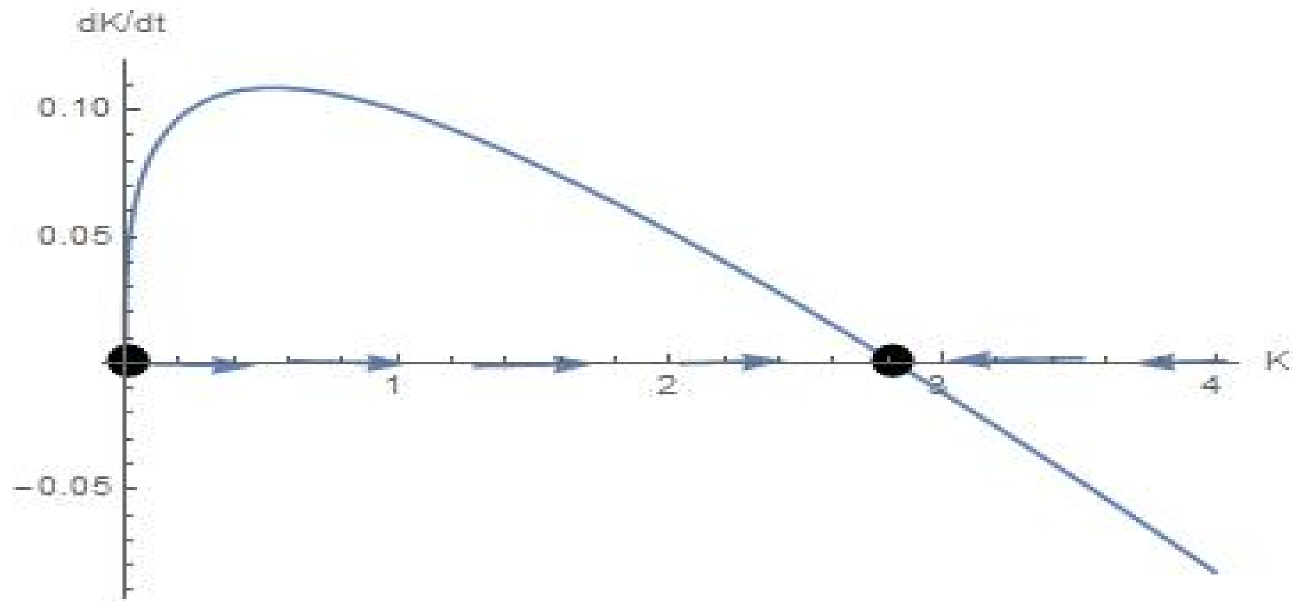
Steady state

Steady State $\Leftrightarrow \dot{K} = 0$

Unstable steady state vs Stable steady state



Steady State (Velocity Interpretation)



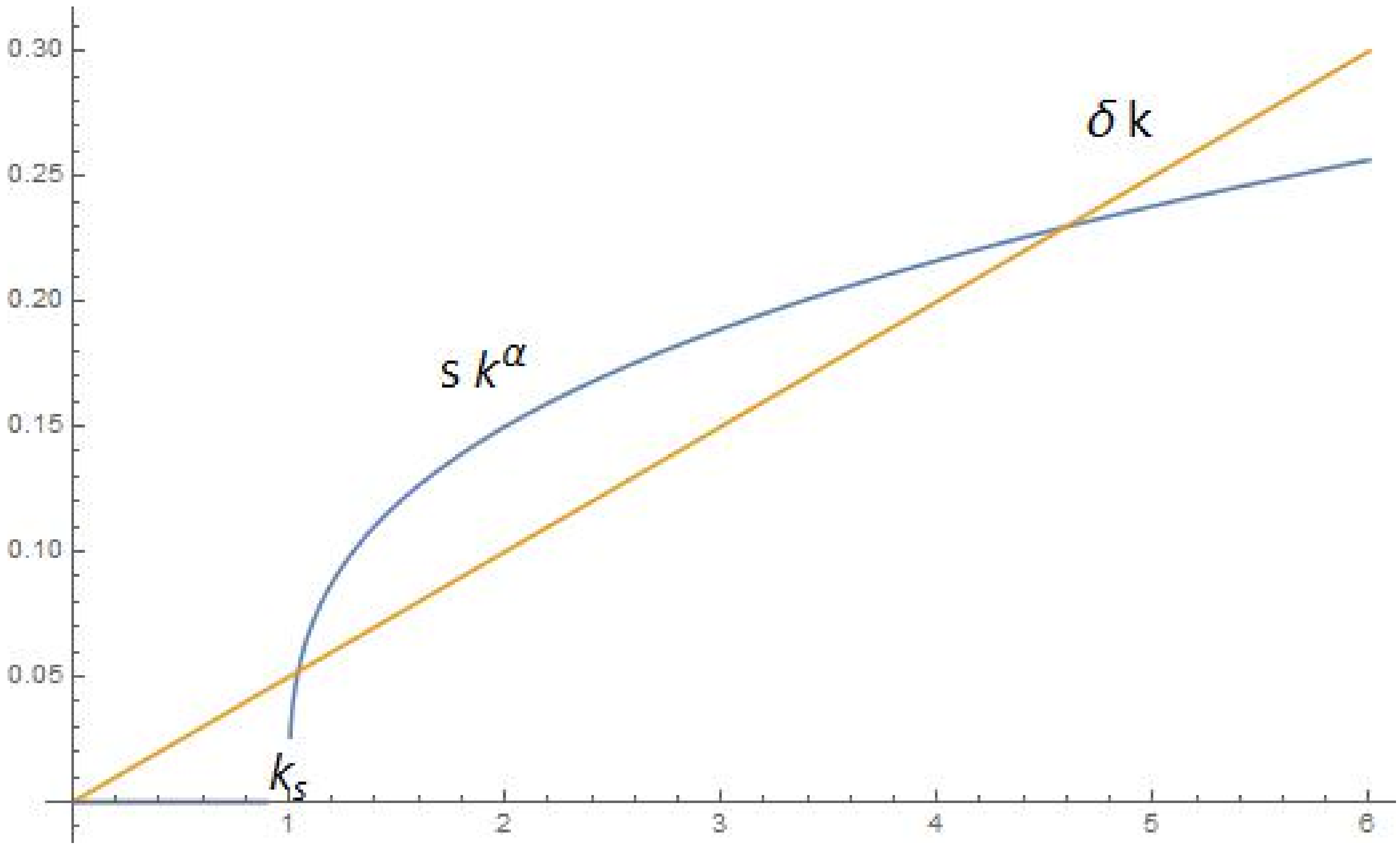
If $\dot{K} > 0$ then go right

If $\dot{K} < 0$ then go left

A Cool Example 1
(Poverty trap)

$$f(k) = \begin{cases} 0 & k < k_s \\ (k - k_s)^{\alpha} & k \geq k_s \end{cases}$$

Example 1 (cont's)



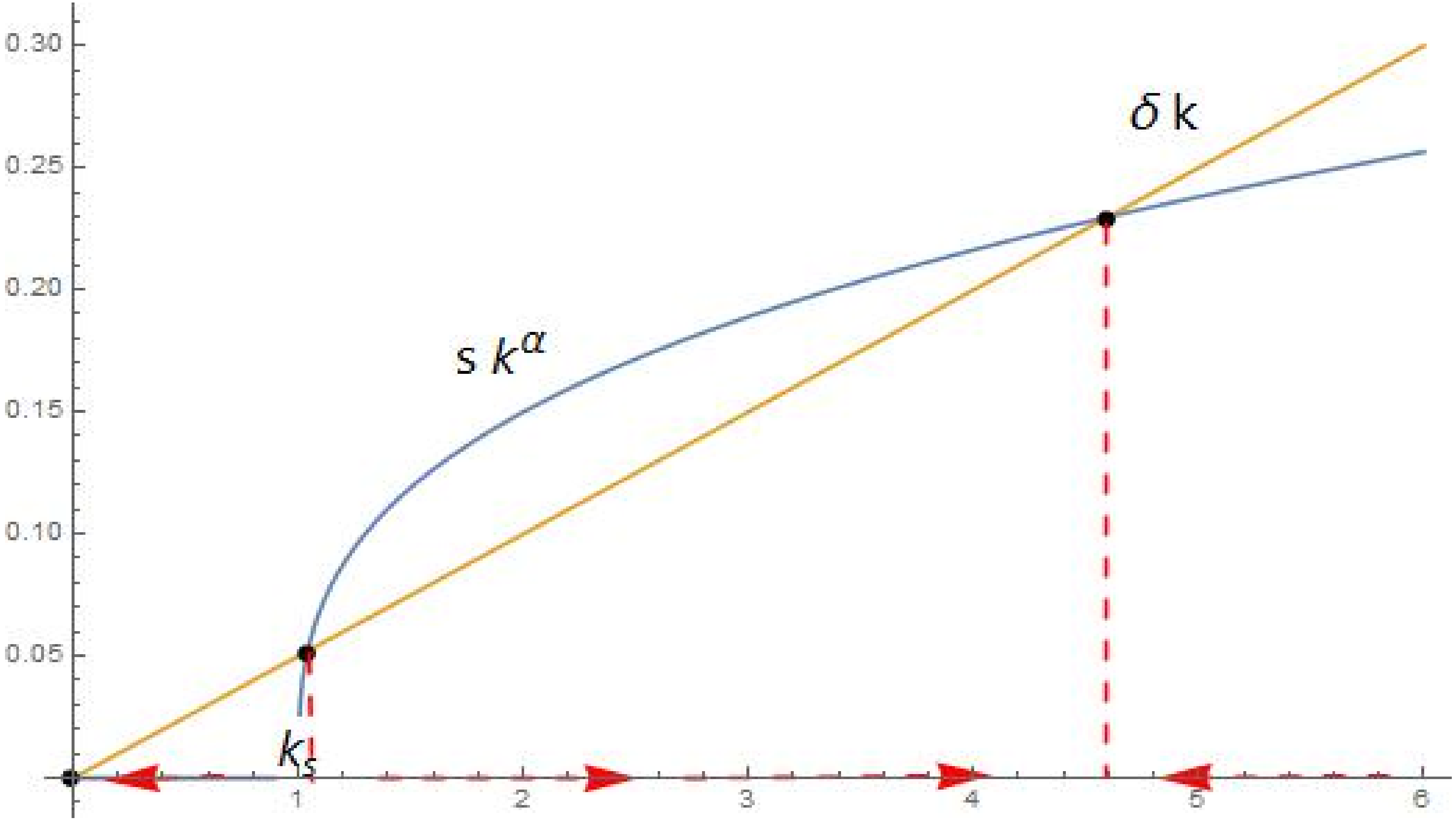
Example 1 (cont'd)
What is going on?

Think about the Inada conditions. Are they satisfied?

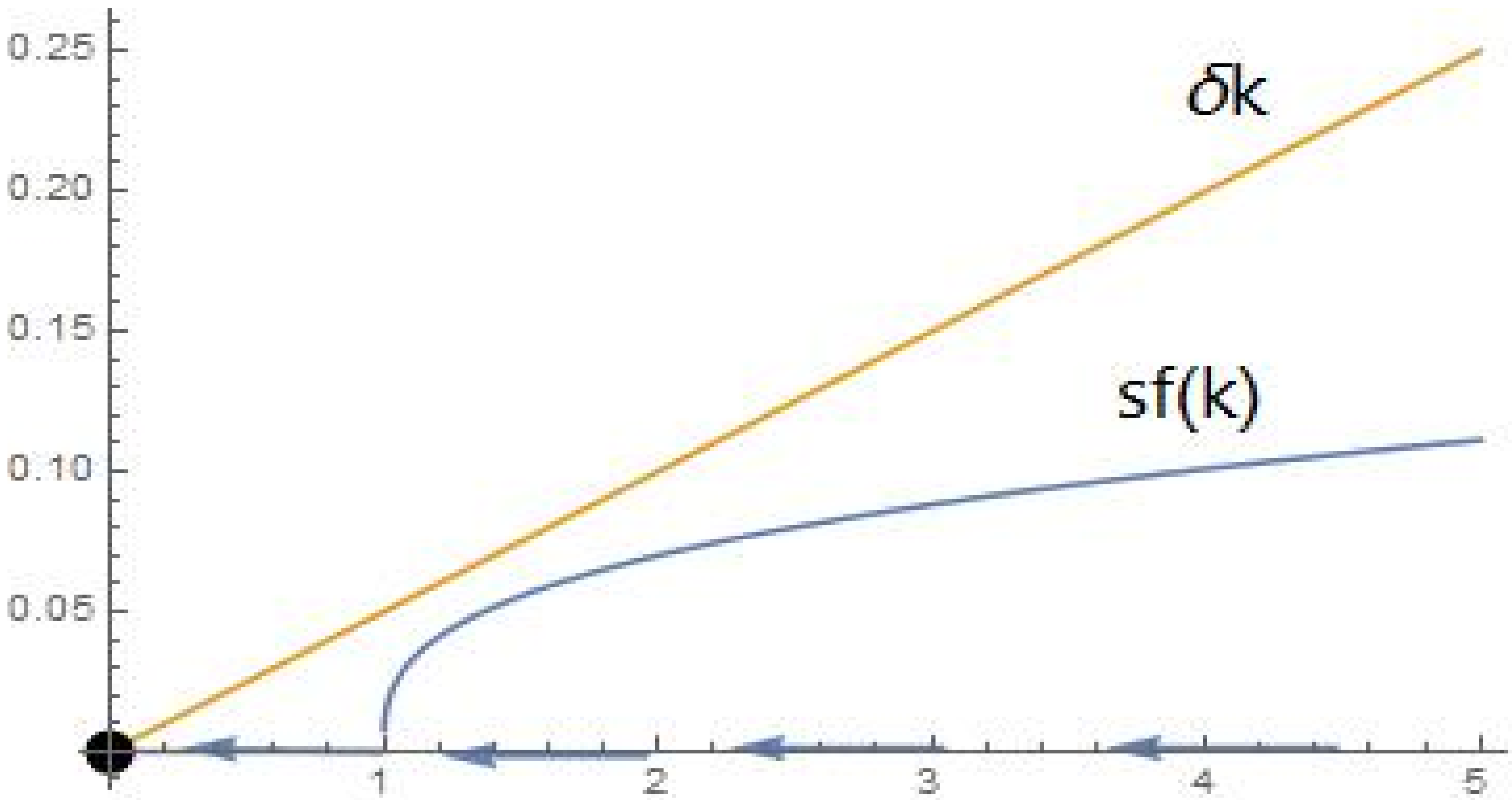
$$\lim_{k \to 0} \frac{\partial F}{\partial k} = 0$$

$$\lim_{k \to \infty} \frac{\partial F}{\partial k} = 0$$

Example 1 (cont'd)



Example1 (cont'd)
(The horrible case)



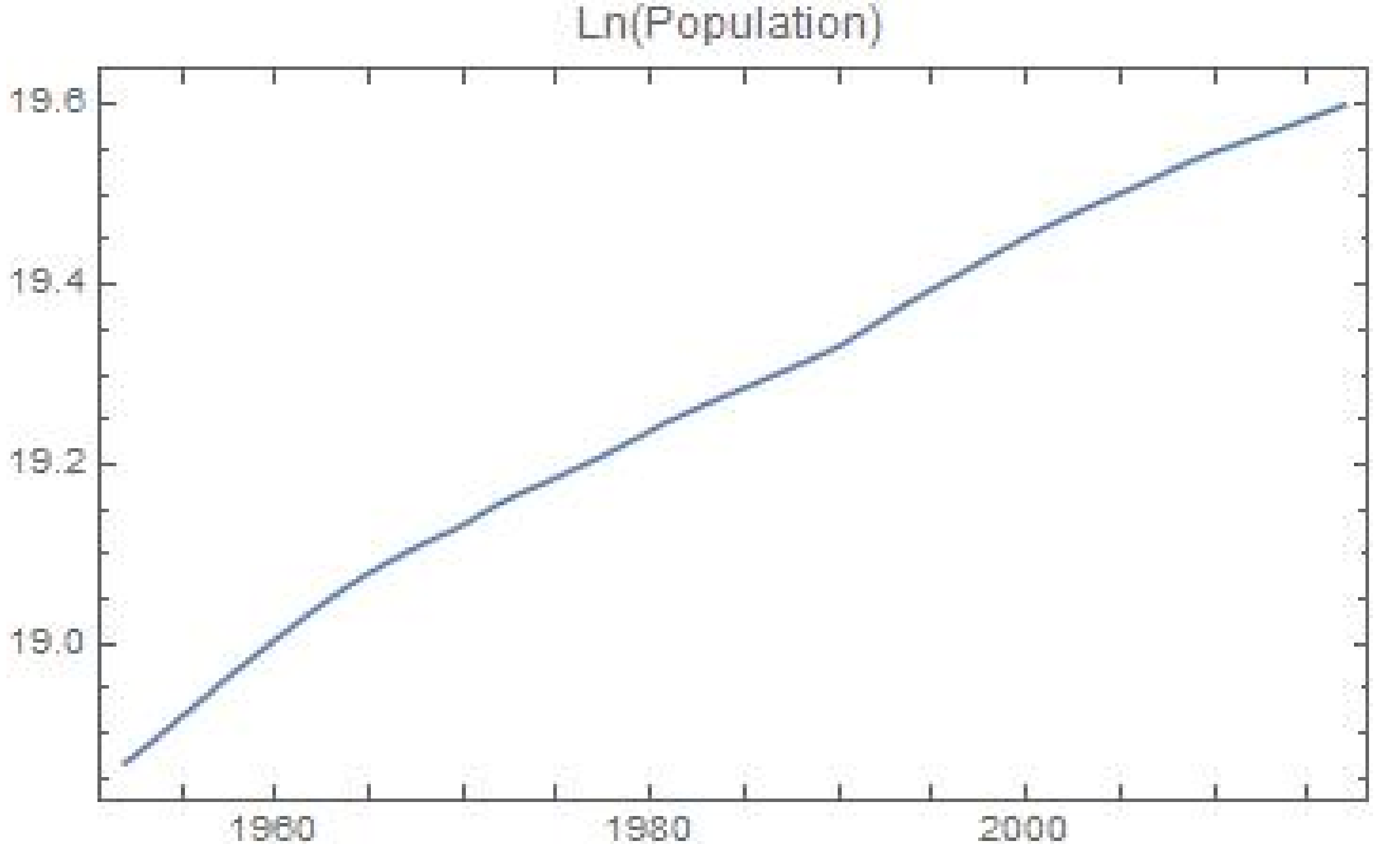
Solow with Population Growth

$$L = L_0 e^{nt}$$

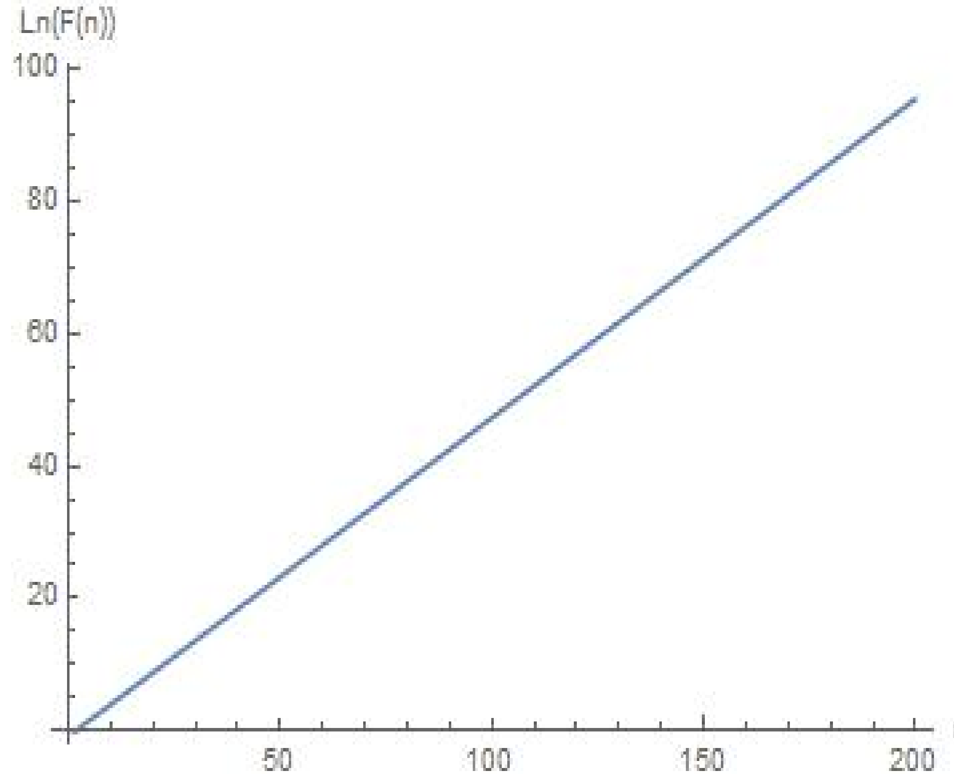
$$\dot{K} = sY - \delta K$$

$$\dot{K} = sK^{\alpha} L^{1-\alpha} - \delta K$$

Labour Growth



A population growth model
1170 – 1250



The new Law of Motion (for Capital)

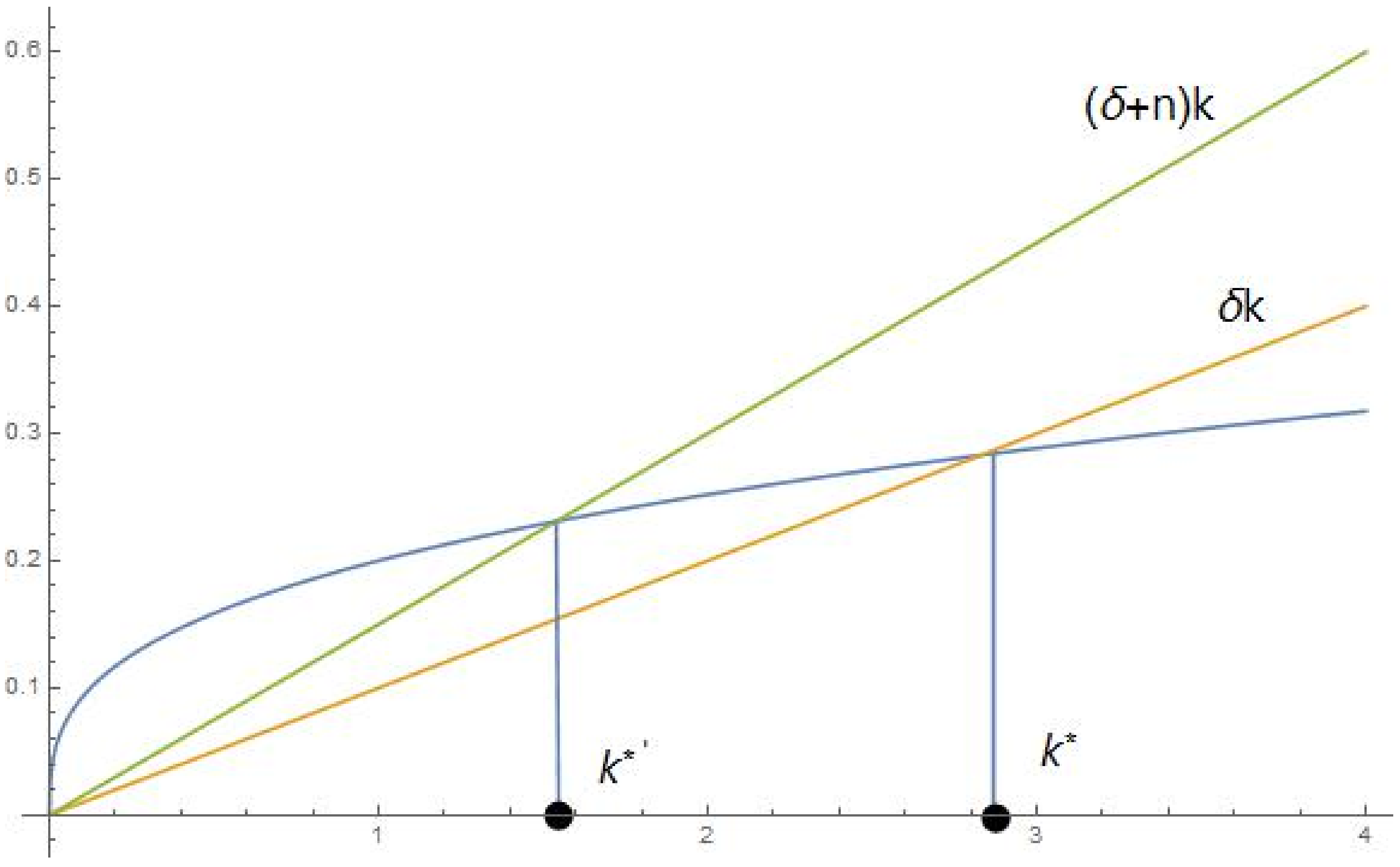
Define:

$$k = \frac{K}{L}$$

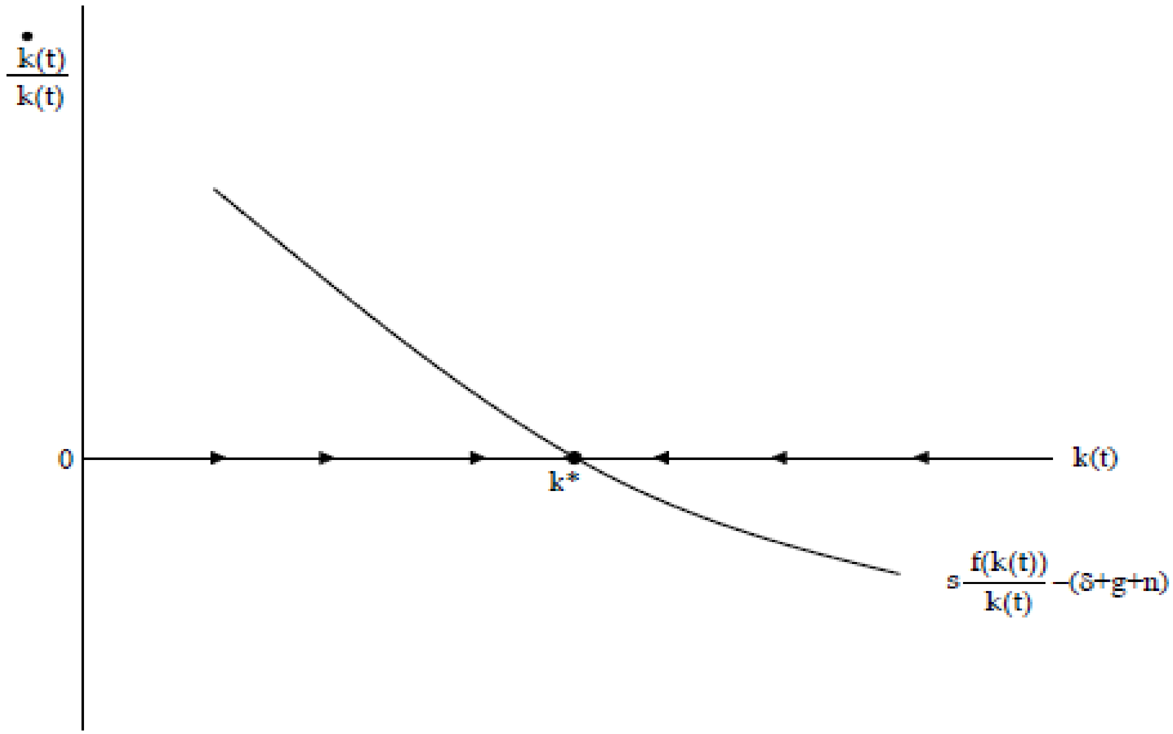
$$\dot{k} = sy - (n+d)k = 0 \quad \Rightarrow sk^{\alpha} - (n+d)k = 0$$

$$\Rightarrow k^{*} = \left(\frac{s}{n+d}\right)^{1/(1-\alpha)} \qquad \text{and} \qquad y^{*} = \left(k^{*}\right)^{\alpha} = \left(\frac{s}{n+d}\right)^{\alpha/(1-\alpha)}$$

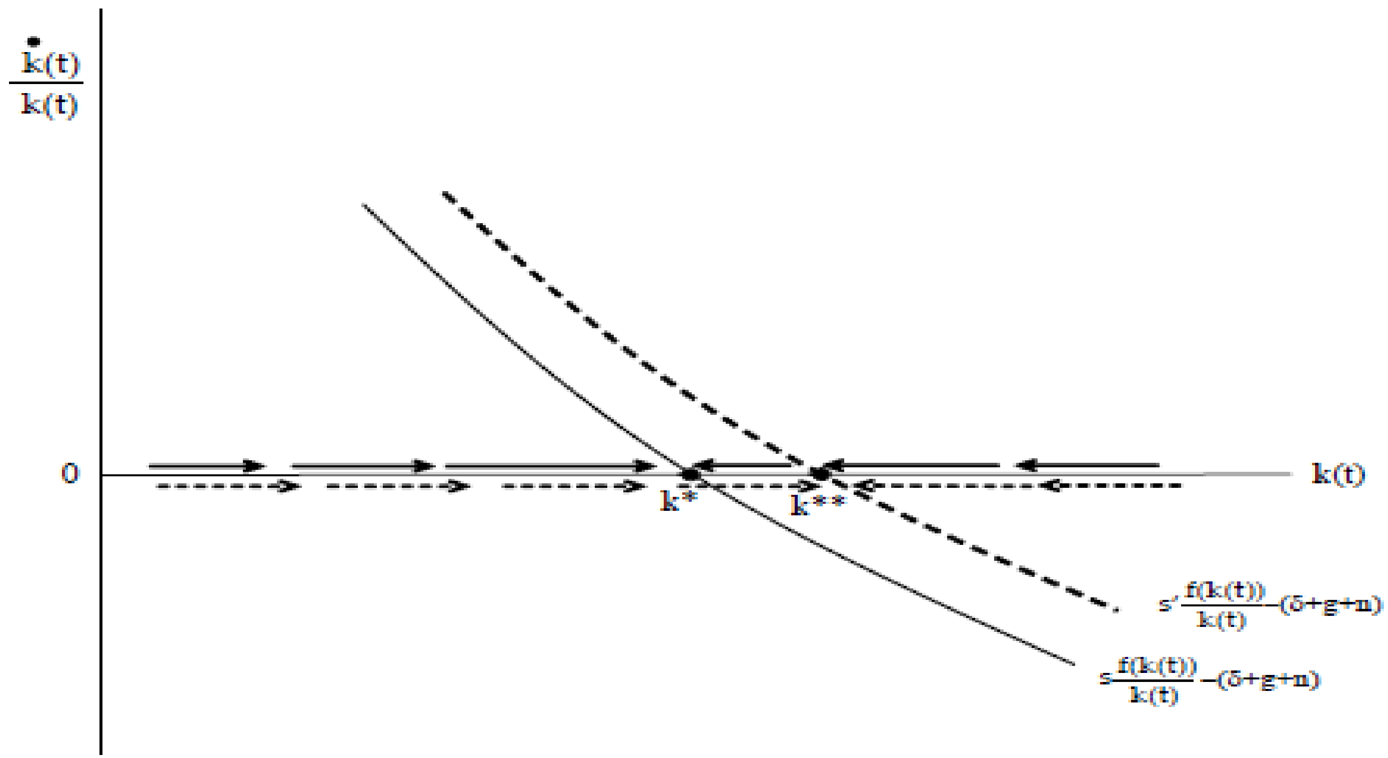
The New Steady State



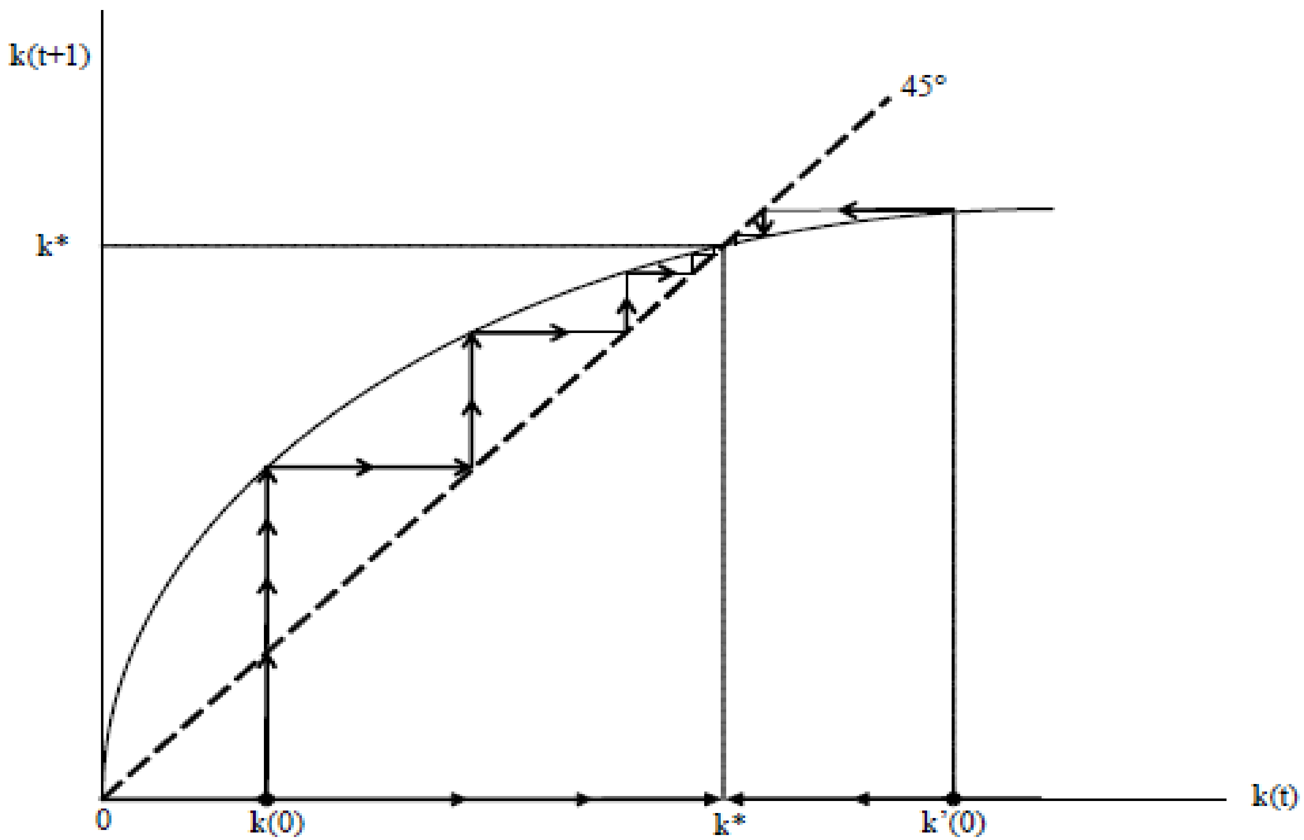
Transitional Dynamics



Transition
(Changes in Saving Rates)



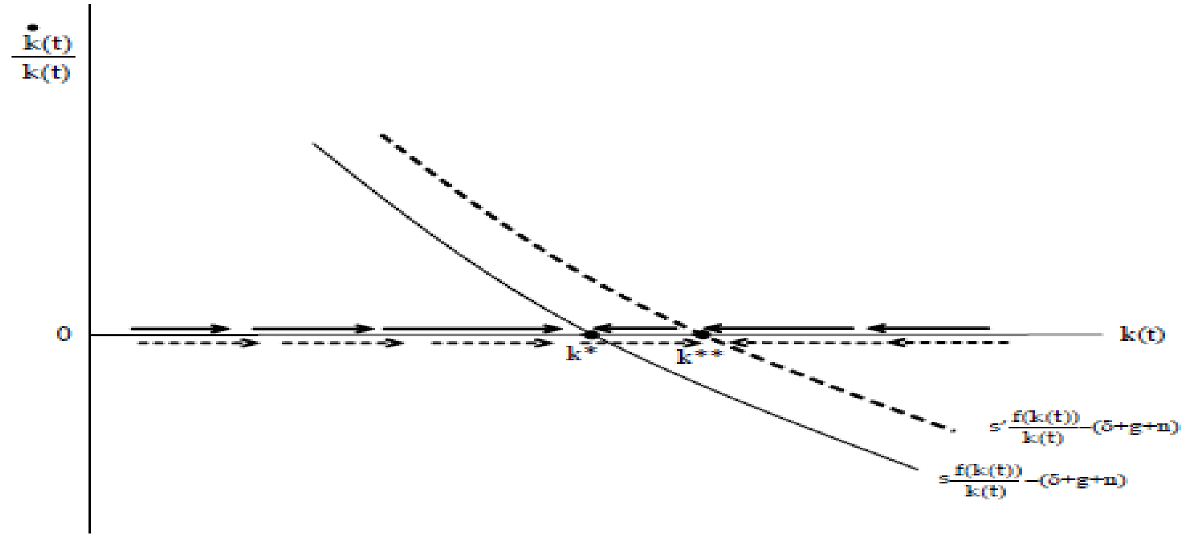
Discrete time



Example 2
(Destruction of Capital)

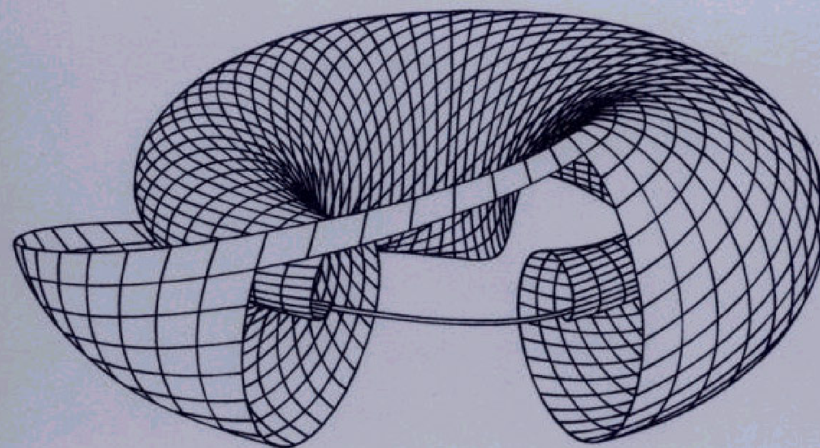
Growth Rate

$$g = \frac{\dot{y}}{y} = \frac{\dot{k}}{k}$$



STUDIES IN NONLINEARITY

NONLINEAR DYNAMICS AND CHAOS



With Applications to Physics, Biology, Chemistry, and Engineering

STEVEN H. STROGATZ